A Comparative Study of Electricity Demand Forecasting Models for Residential Hot Water at the Individual Household Level

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Abstract

Understanding short term future electricity demand is necessary for maintaining a robust and efficient electricity grid. Research on electricity demand forecasting has traditionally ranged in scale from country-wide grid-level down to individual buildings. However, with emerging technologies allowing smart control of appliances, interest in forecasting the electricity demand of individual appliances is increasing. Smart control of appliances can improve energy efficiency and provide beneficial cost-reducing services to an electricity grid.

In New Zealand, electric hot water cylinders which are present in ~90% of households are currently controlled en-mass using ripple-control as part of centralised demand response programmes to reduce demand at peak times. Smart control of individual cylinders based on their forecasted demand could result in a larger reduction in demand with more nuanced control and less chance of households running out of hot water. This thesis compares a number of existing forecasting methods to the problem of forecasting the electricity demand of residential hot water cylinders at the individual household level.

The forecasting models selected for analysis in this thesis were chosen based on their suitability according to existing literature, and preliminary exploratory analysis. A range of conventional forecasting models were assessed including autoregressive integrated moving averages (ARIMA), linear regression, and seasonal and trend decomposition with local polynomial regression (STL). In addition, one artificial intelligence model was assessed, namely support vector machines (SVM). A random walk model was included as a naive benchmark model for comparative purposes.

Existing one minute resolution household electricity demand data that was previously collected in the GREEN Grid study was used for forecasting model development. We selected 22 suitable households from this dataset where the demand of electric hot water cylinders was separately metered. Data was averaged over half-hour periods to mimic data currently available from smart meters in New Zealand. The hot water demand data from each household was separated chronologically into 'training' and 'validating' sets. Models were fitted to the training data and tested against the validating data to prevent inaccurate results from overfitting. The models were compared based on (i) forecast accuracy, (ii) computational speed of model fitting, (iii) interpretability, and (iv) ability to replicate underlying physical processes.

A relationship was discovered between the electricity demand of other appliances in the household and future hot water demand, which was incorporated into some models. SVM models were found to be the most accurate, with 16% lower errors than the naive model, however they performed very poorly in other metrics. The most complex conventional model incorporated STL with ARIMA while including

other electricity demand as an external regressor. This performed almost as well as the SVM models in accuracy, while also performing reasonably well in other metrics, and based on the analysis in this thesis, would be the recommended model for use in smart control.

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Glossary

AC: Alternating current
AI: Artificial intelligence
AIC: Akaike Information Criterion
ANN: Artificial neural network
ARIMA: Autoregressive integrated moving average
ARIMAX: Autoregressive integrated moving average with external regressor(s)
CPD: Congestion period demand
DHW: Domestic hot water
EDB: Electricity distribution business
IR: Instantaneous reserves
HWC: Hot water cylinder
HVDC: High voltage direct current
PEI: Percentage error increase during peak periods, see (3.33)
RMSE: Accuracy metric, “root mean squared error”, see (3.32)
RSS: Residual sum of squares, see (3.5)
STL: Seasonal decomposition of time series by Loess
SVM: Support vector machine

Chapter 1

Introduction

In order to reduce the risk of catastrophic climate change, the International Panel on Climate Change has recommended stabilising atmospheric CO_2 levels at 450 parts per million [1]. In order to meet this target, it is critical we achieve deep electrification of transport and other industries that currently rely predominantly on fossil fuels for their energy supply [2].

Ensuring electricity is cheap and plentiful will assist the shift toward electrification. For this reason, it is paramount we reduce inefficiencies within electricity networks. This research is concerned with two such inefficiencies.

The first inefficiency is that of unnecessary or premature capital expenditure on electricity networks. In the absence of demand management, electricity grids must be built and maintained with the capacity to generate and transmit enough electricity to cover periods of extremely high demand, while in many countries only a portion of this capacity is being used for the vast majority of the time. This is an example of economic inefficiency.

The second inefficiency is that of heat losses from residential hot water cylinders. Many households constantly maintain a large tank of water at around 65°C, while only requiring a portion of it at particular times of day. Even highly insulated tanks experience significant heat loss. This is an example of energy inefficiency.

This chapter explains these two inefficiencies in more detail and outlines the role that smarter control of hot water cylinders may play in alleviating them. It then goes on to explain the value that residential hot water electricity demand forecasting models would bring to such a smart control system. The research conducted for this thesis is focussed on the construction and comparison of a variety of these forecasting models.

The data used to construct and test these models was collected from households in New Zealand as part of a previous study [3]. New Zealand has a high percentage of both electric hot water cylinders and households with smart electricity meters, and therefore would be an ideal candidate for the kind of smart control system envisioned in this thesis. As such, this research is presented from a New Zealand context. However, the implementation of smart control with demand forecasting developed in this thesis would improve economic efficiencies of any electricity network with a high proportion of electric hot water cylinders, and energy efficiencies of hot water cylinders in general.

1.1 Supply and demand of electricity in New Zealand

Following privatisation of the national electricity grid in 1996, the generation, transmission, distribution and retail of electricity in New Zealand is carried out by a number of separate entities. Electricity generation is the process of converting other energy sources, such as heat, solar radiation, or kinetic energy into electricity. While distributed generation such as rooftop photovoltaics (PV) is gaining in popularity, around 99% of electricity generation capacity is currently provided by centralised generators [4]. The New Zealand electricity grid is composed of around 200 points at which high voltage electricity may enter or exit. These are referred to as the grid 'nodes'. Transpower, the systems operator, is in charge of the transmission of electricity, generally from the generators to the grid nodes. Local network companies (also known as electricity distribution businesses or 'EDBs') are in charge of distributing lower voltage electricity from the grid nodes to electricity consumers. Electricity retailers purchase wholesale electricity from generators, and in turn sell this to consumers. In addition, EDBs charge retailers a fixed price per kWh of electricity delivered for their service [4].

The consumption of electricity in New Zealand varies over a number of temporal scales. Throughout summer months, in hot and dry regions, electricity demand on average increases due to the increased use of air conditioners and irrigation systems. Throughout winter months, in cooler regions, electricity demand on average increases due to increased use of electric heaters. These average increases are more pronounced at certain times of day, generally reflected as two 'peaks' in the morning and evening. This is due to the effects of high residential electricity demand during these periods [5].

As electrical energy cannot be stored, electricity supply must match this fluctuating demand. This becomes progressively more expensive to achieve as peak demand increases [6]. In New Zealand, each additional kW of this peak-time demand is estimated to cost around $187 per year in additional costs [7]. The challenges associated with supplying electricity in order to effectively meet demand are likely to be exacerbated by the predicted increase of renewable energy in the grid, as renewable energy generation is often reliant on weather conditions, and consequently can not easily be shifted to meet demand [8].

Failure to match supply and demand may result in grid black-outs, as well as large-scale damage to electricity-using appliances that operate under specific AC frequencies. New Zealand's electricity system frequency must be maintained at or near 50 Hz. In general, when more electricity is generated than is being consumed the AC frequency increases, and when more electricity is being consumed than is generated the AC frequency drops. In the event of a large generator unexpectedly going offline, an HVDC pole tripping, or other large disruptions to supply there must be reserve electricity available so that the frequency is maintained within an acceptable range (between 48 and 52 Hz). Ensuring that there is enough capacity to provide electricity at times of very high demand or during unexpected grid emergencies requires the building and maintenance of generation and transmission infrastructure that sits underutilised for the majority of the time. This is a very costly and inefficient situation [9].

1.2 Demand response

There are currently a range of mechanisms in place to ensure that the supply of electricity balances with varying demand [9], and that currents do not exceed transmission and distribution infrastructure constraints [10]. One method to deal with the imbalance between the supply and demand of electricity is to adjust demand to meet supply. This is referred to as demand response. The effective use of demand response results in a reduction of grid peaks, which delays or defers the need to increase the generation and transmission capacities of the grid. The benefits to the grid associated with demand response are reflected in the ability to monetize load reducing capacity through various electricity markets.

The wholesale price of electricity in New Zealand is determined through supply and demand within a spot market. Generators submit bids on this market to supply electricity to each grid node at a particular cost during each half-hour period. These are then aggregated to form a supply curve for each node. Forecasts are made for the demand at each of these nodes, and the predicted quantity of electricity is purchased from generators to cover this demand [11]. As demand forecasts can not be perfectly accurate, the systems operator must also purchase ‘reserves’, so that if demand is greater than predicted, they can continue to match demand with supply. While these reserves are often in the form of generators running under capacity, they can also be in the form of electricity consumers providing the ability to reduce their demand if necessary. These reserves are bought and sold in separate markets.

An example of such markets is the ‘instantaneous reserves’ markets. Instantaneous reserves markets are national markets concerned with stabilising the grid frequency during times of severe demand peaks or grid emergencies [12]. The systems operator will pay a premium for any electricity that can be made available during these periods, either through generators ramping up latent generating capacity, or, relevant to this thesis, consumers able to shed demand during these periods (known as interruptible load). Interruptible load that is able to be shed within one second and remain reduced for sixty seconds is eligible to be traded in the fast instantaneous reserves (FIR) market. Interruptible load that is able to be shed within sixty seconds and remain reduced until instructed otherwise by the systems operator is eligible to be traded in the sustained instantaneous reserves (SIR) market. Electricity consumers with a large enough reducible load may sell their ability to decrease demand in these half-hour ahead markets. These actors are then paid for providing this service, regardless of whether or not they were required to perform it. Load reduction often comes from large industrial or agricultural consumers who have calculated that it is profitable to provide this service, even with the risk of having to shut down large electricity-using equipment at times. Alternatively, load reduction may come from remotely controlling an amalgamation of smaller electricity using appliances, such as hot water cylinders.

Price variations within these various markets occur according to supply and demand. Prices rise when there is high electricity demand on the grid or there is limited availability of electricity resources with low marginal cost of production, such as those from renewable sources. Conversely, prices fall when electricity is cheap to produce, or total electricity demand is low. These prices fluctuate wildly, with the Benmore node BEN2201 having spot market prices ranging from $0.02/MWh to $894.38/MWh during 2017 for example.

While some larger commercial and industrial electricity consumers are exposed to spot market pricing and may adjust their consumption accordingly, residential electricity consumers are usually charged a fixed price per kWh, with the price variability being 'bundled in' by the retailer. Historically, residential customers were unable to be exposed to these variations in electricity price, as older analog electricity meters only read total consumption of electricity over the billing period. However, in New Zealand there has been significant efforts by electricity retailers and EDBs to deploy advanced electricity meters (known as 'smart meters') to residential electricity consumers. These smart meters collect and transmit electricity demand data that has been averaged over each half-hour interval. The time of use is recorded along with the quantity of electricity used over that period. This provides the capability for residential electricity consumers to be charged prices that vary according to the time of use. As of 2017, smart meters were installed in 62% of New Zealand households [4].

Congestion period demand (CPD) charges are another market-based mechanism for adjusting demand. Large electricity users are often provided a discounted kWh price of electricity, while being charged high penalties for any electricity consumed during peak times where there is very high local network demand (known as 'congestion periods') [13]. These charges are calculated by local network companies, and charged to retailers, who in turn pass the charges on to their customers, the electricity consumer. Once again, this assigns a market value to the ability to shift electricity demand in time. These demand charges can make up a significant portion of the electricity bill of a large consumer, and are a significant source of revenue for EDBs [14]. Currently, residential consumers are not directly exposed to these charges. Instead, the charges are 'bundled in' as a flat rate per kWh lines charge in the electricity bill.

There have been proposals [15] that a market-based tactic to assist demand response efforts would be to encourage residential consumers to move to a real-time pricing plan that uses price signals in order to disincentivise peak-time or otherwise high-cost electricity usage. Electricity company Flick in New Zealand is already providing this, offering customers with compatible smart meters the spot market electricity price (with additional base costs). An application that displays the current spot price is provided to customers, with the intention being that consumers attempt to reduce demand during times of comparatively high prices. While this method of 'deliberate' demand response is well intentioned, studies of consumer response to changes in electricity price indicate that prices generally have little effect on consumption patterns when relying on consumers to deliberately alter their electricity consumption [16]–[18].

1.2.1 Ripple control

One solution to the lack of consumer response to price signals is to automate or remotely control the scheduling of appliances in order to take advantage of price differentials. This already occurs in New Zealand, to an extent, in the form of ripple control. Under a ripple controlled system, EDBs provide a discounted electricity rate for appliances that are separately metered, under the condition that they are able to remotely cease the supply of electricity [13]. This is achieved through sending high frequency signals though the electricity lines to be interpreted by specialised receivers that then suppress the operation of the appliance they are connected to (usually hot water cylinders or irrigation systems). EDBs use this system both to defer upgrades to their infrastructure that would otherwise be necessary, and also as a form of revenue, by selling demand reduction capacity into the national reserves markets previously mentioned. Despite these benefits, ripple control is currently underutilised, with only around 50% of electricity customers connected to ripple control [5]. The ripple control system as it currently operates has been essentially unchanged since its introduction to New Zealand in the 1950's, and there is significant room for improvement utilising more recent technology [6].

1.3 Smart control of hot water cylinders

Appliances that may have their electricity demand times shifted on a daily scale with limited impact to the end user are referred to as 'deferrable load'. Devices such as washing machines and dishwashers exhibit this property, along with battery storage systems in the form of electric cars and bicycles, and some thermostatically controlled appliances such as the focus of this thesis, electric hot water cylinders. The potential to temporally shift their demand for electricity with minimal impact to the end user implies that devices with deferrable load are obvious candidates for autonomous control objectives [19].

With the rapid rise of 'internet of things' technology, whereby devices are interconnected, there is now the ability to remotely control deferrable load to facilitate a number of objectives. Devices may be scheduled around availability of renewable energy sources for example, or optimised to minimise costs through participation in demand response markets. The ability to remotely control deferrable load within an electricity network is part of a new paradigm in electricity distribution known as the 'smart grid'. The smart grid is a power grid where energy producers, network companies and consumers are intelligently connected to each other.

On the consumer side the smart grid is largely enabled by smart meters, which have the ability to store, display and transmit data associated with the electricity being consumed [15]. This two-way information communication is proposed to encourage the integration of distributed (generally renewable) power generation, and allow sophisticated control and automation to proliferate [8]. The process of autonomously scheduling appliances for financial or energy efficiency benefits is referred to as 'smart control'. Many residential appliances have deferrable load, and smart control technology could potentially enable them to provide significant demand shifting [20].

In New Zealand, it is estimated that residential electricity usage contributes around half of peak demand [21], and residential hot water use is around 30% of residential energy used [22]. In New Zealand, electric hot water cylinders (HWCs) are very common, with 91% of households having electricity as an energy source for water heating [22]. A study by the Household Energy End Use Project showed that hot water usage makes up to 36% of residential peak demand [22], with other research suggesting it may be up to 50% [6]. Smart control of residential hot water cylinders is thus particularly promising for demand response, as it is both a deferrable load, and a key contributor to network demand peaks [4].

In addition to assisting demand response objectives, individual control of hot water cylinders can improve their efficiency by only heating as much water as is required by the household. Any object that is hotter than its surroundings will suffer from heat loss, with the amount of heat lost being proportional to the difference in temperature between the body and the surroundings. Thus, as hot water loses heat energy to the environment faster than cooler water, storing hot water in excess of what is required is inefficient. Smart control simulations have demonstrated energy savings of up to 12% in hot water systems that were only heated according to what was predicted to be required [23]. Energy savings could be increased up to 17% if demand predictions were perfectly accurate [24]. The ability to store only as much hot water as will be required by the household (with the remainder of the tank being left as cooler water) therefore has the potential to reduce total hot water electricity consumption.

1.4 Predicting electricity demand

Accurate individual demand predictions would allow devices to be controlled according to the usage patterns of individual households [25]. This is beneficial within a smart control system as it would provide additional capacity for control while reducing negative effects on service [26]. It would also improve the ability to participate in half-hour ahead demand response markets, as demand predictions allow more accurate estimations of the capacity for load-shedding.

Following the roll-out of smart meters in New Zealand, there is now the ability to collect and process essentially 'real-time' data on hot water electricity consumption for individual consumers. This allows ripple control (or a similar system) to be personalised to individual household usage patterns, with intelligent control systems including predicted demand in their algorithms.

There are many different techniques whereby key aspects of the processing, analysis and extrapolation of data can be carried out algorithmically by a computer. Different techniques use different algorithms to carry out this process, some of which use more computational resources than others. This is most intuitively understood as the amount of time a computer with a particular processing capability will take to construct the forecast.

Techniques commonly used for predicting hot water electricity demand may be broadly separated into the categories (i) physical modelling, (ii) conventional forecasting, and (iii) artificial intelligence (AI) [27].

1.4.1 Physical modelling

Smart control of residential hot water cylinders is an active research topic [28]–[32]. Much of this research uses computer models that mathematically represent the physical behaviour of a hot water cylinder and its operation under standard residential hot water demand patterns [28]. These models may then be used to simulate the impacts of smart control, as well as provide forecasts of electricity demand.

In order to physically replicate water flows within the model, generalised hot water demand patterns must be input as model parameters. These demand patterns are often based on simple statistical or mathematical relationships related to time, with some artificially generated 'randomness' for added realism [28]. Uncovering the underlying patterns within data that can be represented as simple statistical or mathematical relationships is the primary focus of conventional forecasting methods.

1.4.2 Conventional forecasting methods

Rather than building a predictive model based on prior knowledge of physical characteristics of a system, conventional forecasting methods make predictions based on historical time series data. Time series data involves measurements of variables sequentially or at fixed intervals of time [33]. The time intervals between each measurement are known as the data 'resolution'. Time series data that has measurements recorded every hour, for example, would have a one hour resolution. Time series data may be processed, analysed and extrapolated in a manner that produces predictions of future events or occurrences, a process known as 'forecasting'.

Conventional forecasting methods involve discovering underlying statistical properties of time series data. These methods often require significant human input, generally requiring an initial data analysis, testing, and trial and error. However, once an optimal conventional model is discovered, it can generally be constructed with reasonable computational speed. In addition, the insights that are gained from conventional forecasting methods may be applicable to other research such as physical modelling.

There are two distinct conventional time series forecasting techniques utilised in this research; regression analysis, and time series analysis. Regression analysis utilises data of separate but related (explanatory[1]) variables and makes predictions based

[1] Multivariate data can be thought to be made up of both dependent variables and explanatory variables (also known as 'independent variables' or 'regressors'). Broadly speaking, dependent variables are those we are interested in measuring or predicting (in this research, hot water electricity demand). Explanatory variables are those we expect to impact our dependent variables (in this

on basic statistical relationships between them [34]. For example, if there is the ability to know (or sufficiently accurately predict) the value(s) of the independent variable(s) at a particular future time, then a forecast for the dependent variable may be calculated at that same time based on the causal relationship between them [27]. In the context of electricity demand forecasting, relevant predictable independent variables may be social factors such as public holidays, or (accurately forecasted) weather conditions [35]. However if there is no obtainable value(s) for the independent variable(s) at the future time for which a prediction is required, more subtle techniques must be employed. One option is to establish a relationship between the value(s) of the independent variables and subsequent values of the dependent variable. In the context of DHW electricity demand forecasting, this may be the relationship between the electricity demand of other appliances, and subsequent values of DHW electricity demand.

Another conventional forecasting method known as 'time series analysis' utilises patterns in the time series data of the variable being predicted. One important technique used in time series analysis is the separation of the data into individual components, namely trend, cycles and 'noise'. In the context of DHW electricity time series data, the trend would be a gradual increase (or decrease) in average hot water use over long periods of time. Cycles would be average daily use patterns such as morning and evening peaks previously mentioned. The noise is any deviation of the data from the combination of trend and cycle components (people not actually using hot water according to strict mathematical relationships). This method, sometimes referred to as 'decomposition', can be achieved using a technique called 'seasonal decomposition of time series by Loess' (STL), and is explained in more detail in Section 3.4.2.

Additionally, forecasts may be made by understanding the relationships between a variable and its previous values. Autoregression (AR), moving average (MA), and a combination of the two (ARIMA) are techniques commonly utilised in electricity demand forecasting to obtain these relationships [24], [25], [36], [37], which are expanded upon in Section 3.7.

Conventional forecasting techniques are generally prized for being simple and interpretable, while requiring minimal computing resources. However, they often require significant human input. In addition, they are often outperformed in accuracy by more advanced AI methods.

1.4.3 AI forecasting methods

Artificial Intelligence (AI) methods are those that use more mathematically complex techniques from the field of machine learning for forecasting. The distinguishing feature of these methods is their ability to process and establish relationships between data autonomously. This allows them to be constructed with minimal human input or pre-existing insight into features of the data. This is done through a method known as 'training'. To train an AI model, an algorithm iteratively adjusts parameters within

research we use time and the electricity demand of other appliances).

the model until the outputs (predictions) of the model best match the actual data, under certain prespecified constraints.

The comparatively autonomous nature of AI methods does not necessarily make them inferior, as they still have the ability to obtain (often) highly accurate predictions. There is a trade-off however, as their outputs tend to be 'black box', generally providing less in the way of understanding underlying processes than many conventional forecasting methods. In addition, AI modelling can be hindered by the large amounts of computing resources necessary to carry out their training.

Two AI models commonly used in electricity demand forecasting, namely artificial neural networks (ANNs) and support vector machines (SVMs), are explained in more detail in Sections 2.2 and 3.7.12.

1.5 Aims of this thesis

In this thesis, I aim to

carry out a comparative study of models for predicting individual household hot water electricity demand for smart control systems using data available from New Zealand smart meters.

New Zealand smart meters provide electricity demand data at 30 minute resolution and usually have hot water demand metered separately from the demand of other appliances. Both hot water electricity demand and other appliance electricity demand are included as inputs within these forecasting models. A selection of conventional forecasting models will be explored and compared against both a simple benchmark model and a more complex AI model. Comparative metrics take into consideration both the suitability for incorporation into smart control systems, and for incorporation as demand simulation into physical models. In particular, models are compared based on (i) forecasting accuracy, (ii) computational speed of model fitting, (iii) interpretability, and (iv) ability to replicate underlying physical processes. The research conducted for this thesis uses existing electricity demand time series data that had been collected from New Zealand households in a previous study [3].

1.6 Thesis structure

The structure of this thesis is as follows. In Chapter 2 we present relevant literature. This focuses on key research on hot water modelling, smart control of appliances, and electricity demand forecasting. Chapter 3 presents the methodology used in this thesis. It introduces the electricity demand dataset utilised in this work, and explains the data cleaning and preparatory process. It then describes the exploratory data analysis techniques, the forecasting models utilised, and the metrics by which the models are compared to one another. Chapter 4 presents the results obtained from the preliminary data analysis. It also briefly discusses the implications these results have for selecting appropriate forecasting models. Chapter 5 presents the results from a selection of forecasting models used to predict hot water electricity demand of each individual household in our dataset. The discussion in Chapter 6 provides comparisons of the models based on the metrics outlined in Chapter 3, with optimal models for different purposes presented. It also clarifies the role that results from this thesis may play in a smart control system. A summary of the background and aspirations of this thesis and its results are provided along with potential further work in the conclusion (Chapter 7). Relevant tables, including model parameters and data attributes developed in this thesis are provided in the Appendices.

Chapter 2

Literature review

This chapter provides an overview of the literature relevant to this thesis. While the research conducted for this thesis did not involve any smart control, its aims are best understood through a smart control lens. As such, this chapter begins by noting key literature relating to smart control of appliances, and any relevant findings. Section 2.2 goes on to briefly reiterate how forecasting models are constructed, and then lists a selection of models commonly used in electricity demand forecasting. It then notes other work where models were compared with one another, and the results of these comparisons. Following this, Section 2.3 presents key literature regarding domestic hot water simulations. A selection of physical hot water models used in research simulations are described, along with an explanation of their interrelation with demand forecasting models. Finally, Section 2.5 examines existing research most similar to this thesis. In particular, [26] inspired much of the research in this thesis. A number of its methodologies were replicated, and its key findings were built upon in this work. To conclude this chapter, differences between [26] and this thesis are made clear.

2.1 Smart control

Smart control research generally involves estimating the benefits associated with scheduling or otherwise adjusting the operation of suitable appliances at key times. This control is performed in order to meet a number of objectives, including:

- reducing grid peaks [23], [29], [31]
- providing grid-beneficial services (instantaneous reserves) [38]
- minimising costs under time-varying electricity prices [23], [31], [39]–[42]
- utilising low-carbon electricity [43], [44]
- maintaining consumer comfort under budget constraints [45]
- improving energy efficiency [23], [24]

Insulated hot water cylinders are an example of deferrable load, and as such are good candidates for smart control [4], but similar objectives can be obtained with other thermal devices such as heating and air conditioning systems [40], [45]. This is due to the ability of thermal energy to be stored, with only slow dissipation, providing insulation is adequate. In addition, other devices such as electric vehicles and clothes washing machines/dryers may be used for smart control purposes [39], [41], as they also display some temporal flexibility around their electricity demand.

In order to effectively schedule appliances under smart control, reasonable estimates must be made as to when they are required to be utilised. An electric vehicle, for example, must be sufficiently charged before the next use, clothes must be washed and dry in time for their next use, and a hot water cylinder must be sufficiently hot before the next shower. Accurate predictions of these 'next' usage events provide a smart control system the ability to fine tune their optimisation algorithms to take full advantage of appliance load deferrability without negatively impacting the end user. As such, demand forecasting of appliances in some form is common in smart control literature [23], [29], [31], [32], [36], [41], [42], [44], [45].

2.2 Electricity demand forecasting

In general, forecasting models are constructed using existing data of the variable being predicted, and, in some cases, other relevant data (regressors) that span the same period of time. The models are 'fitted' to the existing dataset such that discrepancies between the existing data and the model are minimised. Once this is completed, these models may then be projected forward in time to where no data exists in order to make predictions. Models of this type are often fitted to electricity demand data (and any relevant regressors) for the purposes of predicting electricity demand.

While much research has been conducted on predicting grid-level [46], [47], substation level [48], or building level [37], [49]–[51] demand, there is increasing interest in predicting the demand of individual household appliances [36], including that of hot water cylinders [24], [25], [52].

When forecasting at grid level, research is generally focussed on effectively matching production and distribution [46], [47], [53]. At building level, the prediction of energy consumption patterns are used when budgeting, negotiating and purchasing electricity contracts, and to detect electricity theft or fraud [49], [53]. As mentioned in Section 2.1, appliance or equipment level electricity demand forecasting is often utilised in smart control research. Additional purposes for appliance level forecasting include the detection of faults, and the minimisation of operation and maintenance costs [53].

There is currently no consensus on the 'optimal' method for forecasting electricity demand. Conventional techniques that are commonly used are either some form of time series analysis such as ARIMA [25], [36], [37, p. @Denis2019], regression analysis [54]–[57], or a combination of the two [46].

There are also a number of 'AI' methods used in electricity demand forecasting. These include:

- Artificial neural networks (ANNs) [58], [59]
- Support vector machines (SVMs) [47], [52], [60]
- ensemble bagging trees [61]

- particle swarm optimisation [31]
- reinforcement learning [62]
- cluster analysis [55]
- dynamic Gaussian processes [58]

ANNs and SVMs are the two most common AI methods used for electricity demand forecasting [27], [50], [63]–[65]. ANNs attempt to mimic biological learning mechanisms by processing a set of training data through clusters of artificial neurons, each of which individually receive (numerical) inputs, and assign them certain weights. These weighted values are then input into other artificial neurons, which perform the same task, until eventually producing a final output. The weights assigned by each neuron are then iteratively adjusted in order to minimise the error of the final output. For further details regarding ANN models for forecasting electricity demand refer to [65].

Despite being in common use, literature suggests that ANNs tend to be outperformed in both accuracy and computational speed by support vector machines when used for electricity demand forecasting [50]. For this reason, they were not included in this comparative work, with support vector machines instead selected as the comparative AI method.

While much electricity demand forecasting research only considers a single model, some work has been done that compares a selection of models with one another. Research comparing AI methods showed SVMs tend to outperform ANNs in accuracy [50] while having reasonable computational speed [50], [65], whereas ANNs in particular were noted as having high computational time [58]. While a comparison between AI methods in [49] showed that an ANN provides slightly more accurate forecasts when predicting a full day ahead, the SVM was most accurate when predicting the next hour. Some comparative research considers both AI and conventional methods [56]–[58], [66]. SVMs were compared against ANNs and linear regression in [56], [57], and found to be the most accurate model. SVMs were not considered in [58], which instead found an AI forecasting method called dynamic Gaussian processes to be by far the most accurate when predicting grid-level electricity load. Dynamic Gaussian processes however were noted in [51] as being unsuitable for building level demand forecasting, and thus, we may assume, also for appliance level forecasting.

Time series and AI forecasting methods were compared against a naive benchmark model in [66] to forecast the electricity demand of office buildings. The effects of including additional independent variables (regressors) to these models was analysed. The regressors chosen were (i) hour of day, (ii) hot water demand, (iii) luminosity, and (iv) workday/not workday. The addition of the regressors increased accuracy drastically in the AI methods, and slightly in the time series methods. Overall, the time series methods had the highest accuracy, although with the addition of the external regressors to the AI method this advantage was slight. A comprehensive review of data-driven building energy consumption prediction studies [64] shows SVMs consistently outperforming ANNs in accuracy for demand forecasting, which is confirmed by another review in [27].

Comparative research in [26] considers a number of time series analysis methods for forecasting volumetric residential hot water demand. These models were fitted to volumetric hot water data collected from 95 UK households. Similar to the research undertaken for this thesis, they also used the `R` programming language, and used simplistic models as benchmarks by which to compare other models to (see Sections 3.7.1 and 3.7.2). They explored a number of time series models for prediction, including exponential smoothing, ARIMA, seasonal decomposition by local polynomial regression (STL), and combinations of these. Predictions were made for hourly intervals up to 24 hours ahead, and models were compared against one another for accuracy. Their most accurate model, a combination of STL and ARIMA, achieved prediction accuracy more than 50% above their benchmark models. Forecasts were also constructed for aggregated demand data, that is, the demand of all the households added together. Accuracy of all models considered was improved when aggregating data in this manner.

Some demand forecasting models, particularly those using conventional forecasting methods, are reasonably easy to interpret. Once they have been fitted to the data, their parameters can be extracted to provide statistical insights into the behaviour behind the data. This is valuable for building 'physical' domestic hot water computer models, as their parameters can be used to construct more accurate simulations of hot water demand, a necessary component of these models [25].

2.3 Domestic hot water modelling

As a physical roll-out of smart controllers would be prohibitively expensive for research purposes, much existing smart control research uses simulations to determine the potential of smart control [24], [28]–[30], [32], [38], [42], [44], [45], [52], [62], [67], [68]. In the context of researching a physical system under smart control, simulations are created using known physical and statistical properties of the system. Physical properties are determined through an understanding of the physics of the system. Statistical properties tend to be determined using existing data collected from the physical system, or relevant subsets of the system. Simulations then use these properties to create (simulate) new data that mimics that which would be obtained if this system was constructed in reality. Parameters of the system being simulated are able to be altered to see how this affects the outputs being studied [28].

Domestic hot water modelling refers to the process of building simulations of a residential hot water cylinder and its operation. In the wider context of computer modelling, these are referred to as 'physical' models. While all models are less accurate than the systems they replicate, analyses that utilise computer models have a number of crucial benefits over real-world alternatives. Firstly, a well-constructed physical model provides the ability to make an analysis regarding alterations to the physical system that may be difficult or expensive to carry out in reality. Secondly, physical models provide the ability to 'speed up' the passing of time, and to be run 'into the future', two features currently unavailable when conducting real-world analyses. Hot water cylinder models may be utilised for a number of applications. Some represent the physical system, with all its temperature gradients and heat and fluid flows, to a high degree of accuracy [69], [70]. These are suited for analyses as to the effects of making physical changes to the system, such as adjusting the position of the outlet [69], [70] or cylinder dimensions [70]. However, a high degree of physical accuracy comes with high computational cost. The more physically simplistic models are more suitable for analyses that span over longer periods of time such as weeks, months or years, as is necessary in simulations of smart control scenarios.

Generally speaking, a domestic hot water model will consist of both a mathematical description of the physical system, and a mathematical or statistical description of the flow of hot water out of the tank, that is, hot water demand. A mathematical representation of the physical cylinder must, at a minimum, take into consideration the internal temperature effects of inflows of cold and outflows of hot water, heat input into the cylinder from the element, and heat loss from the walls of the cylinder. Water temperature(s) within the cylinder are important, both for heat flow calculations, and for determining if and when the cylinder has run out of hot water. In a cylinder of warm water at equilibrium a temperature difference between the top and bottom of the cylinder naturally arises. This can be described in terms of three distinct volumes of water. At the bottom of the cylinder, there is a volume of cooler water, with only a slight change in temperature from the bottom to the top of it. At the top, a volume of warmer water, again with only a slight vertical temperature gradient. These are separated by a thin volume of water known as the thermocline, whereby a rapid change in temperature occurs between its top and bottom [28].

Some models choose to simplify this heat distribution within the tank, considering it instead to be one well-mixed, single temperature volume of water [30], [71], [72]. This method has the benefit of being computationally inexpensive, allowing for temporally extensive analyses to be carried out, to the detriment of some physical accuracy. Other models simplify the heat distribution as two well-mixed bodies of water at the top and bottom, separated by a thermocline of negligible width [28], [73], [74]. While requiring more computer resources than the single-volume models, the two-volume models still a provide relatively computationally inexpensive means of analysing hot water usage, while capturing some additional physical characteristics such as the rapid ‘running out’ of hot water. A schematic of a two-volume model is provided in Fig. 2.1 Finally, some hot water cylinder models use computational fluid dynamics software to maintain very high amounts of physical accuracy, essentially allowing the determination of water temperature at any point within the cylinder [69], [70].

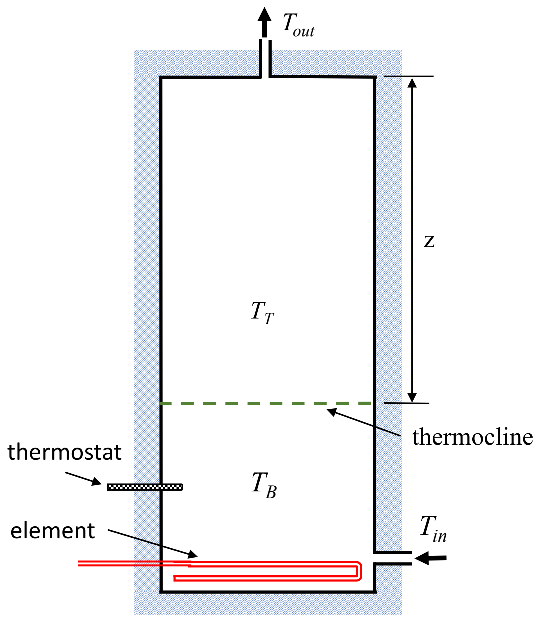

Figure 2.1: A schematic representing a two-volume physical model of a hot water cylinder [28].

Within these models there must be the ability to simulate the demand of hot water. The model in [71] simply schedules three 35L, 38° hot water events at three separate times through the day. The model in [28] simulates domestic hot water usage by assuming up to three major use events (representing showers, bathing etc.) that are random and normally distributed about fixed times of day, with additional minor usage events (representing hand washing etc.) having equal probability of occurring between specified intervals at other times. A similar method was used in [75], however within this research use events were separated into four different categories, each with different (Gaussian distributed) probabilities of volume and time of occurrence. For much of the hot water cylinder research previously mentioned however, the methodology around demand simulation is not clearly elucidated.

Statistical insights gained from parameters within appropriate demand forecasting models have the potential to be incorporated into physical models to simulate demand. This would improve how accurately these physical models represent reality [25], in turn improving accuracy of results from smart control simulations. The research conducted for this thesis does not use any physical models. However, when the forecasting models that are used in this thesis are compared to one another, suitability for incorporation into a physical model is considered (see Section 3.10).

The use of physical models alongside forecasting models in a smart control algorithm has been previously researched in [23]. Residential hot water demand forecasts with Austrian electricity spot price data were provided as inputs in a smart control simulation of hot water cylinders. This was optimised to arbitrage spot market electricity prices and/or reduce overall energy demand. Cost savings of up to 12% were demonstrated when optimising for price arbitrage, and energy savings of up to 12%

were demonstrated when optimising for energy efficiency.

2.4 Model selection

Every forecasting model has unique strengths and weaknesses that render it more or less suitable to the aims of this thesis outlined in Section 1.5. Based on suitability to meet these aims, some models encountered within existing literature were selected for further investigation, while others were disregarded.

This research places emphasis on the ARIMA and related models, as existing literature suggested these would have a reasonable performance over all the comparative metrics (detailed in Section 3.10). For the same reason, SVMs were chosen as a representative AI method. ANNs were not investigated in any depth, as they appeared to require high performance computational resources to execute in a timely manner while not offering accuracy improvements over SVMs. We are interested in predictions based only on data that would be available from any smart meter with a separately metered hot water cylinder, as is commonly available in New Zealand today. For this reason, models that rely on the availability of external data were not utilised, although some regression was carried out using other appliances as the regressor variable.

This chapter indicated that much groundwork has been done in the existing literature as to which models would be good candidates for predicting residential hot water electricity demand. A selection of promising models were chosen for comparison based on their performance and/or prevalence in existing literature. These are described in more detail in the methodology chapter that follows.

2.5 Relationship between this thesis and existing literature

In this section, we compare the aims of this thesis (see Section 1.5) to the existing literature.

While there is some existing research that compares electricity demand forecasting models, comparisons generally only consider accuracy of predictions. When computational costs are mentioned, they are qualitative, and given little focus [64]. There is very little consideration within existing literature to utilising parameters from forecasting models in order to simulate demand within a physical model [25]. While the researchers in [66] use hot water demand as a regressor to improve prediction of total building electricity demand, no research was found that used the electricity demand of other appliances as a regressor to improve prediction of hot water electricity demand.

To the best of our knowledge, there is no existing literature that compares a range of models to forecast residential hot water electricity demand. Within the existing literature, the piece of research most similar to that conducted for this thesis is that of Gelažanskas and Gamage in [26]. While this thesis draws on the key findings within this research, it differs in a number of ways, which are shown in Table 2.1.

Table 2.1: Comparison between this thesis and the most similar piece of existing literature

	This thesis	Gelažanskas and Gamage
Focus of study	Domestic hot water	Domestic hot water
Data resolution	Half hour	1 hour
Data type	Electricity demand	Volumetric demand
Forecast length	Half hour	24 hours
Forecast methods	Time series, regression, AI	Time series
Comparative metrics	Various	Accuracy

Chapter 3

Methodology

This chapter begins by giving an introduction to the existing dataset that was used for this research. It goes on to describe some necessary cleaning and preparation of the data in Section 3.2. In any data analysis, there are preliminary processes that should be undertaken in order to get a general overview of the data [33]. This allows for an informed opinion to be made as to the best techniques to achieve the desired objectives of the analysis. These preliminary processes explore our dataset for the purpose of ascertaining any patterns and attributes that may assist in building our forecasting models. An introduction to terminology and concepts that are fundamental to these preliminary processes is provided in Section 3.3. Analyses relating to the temporal behaviour of the hot water electricity demand data are introduced in Sections 3.4 and 3.5. An analysis regarding the correlation between hot water and other appliance electricity demand is introduced in Section 3.6. Mathematical descriptions of the forecasting models utilised in the main body of work are then introduced in Section 3.7, with applicability to the context of hot water electricity demand forecasting provided. Finally, the metrics by which the models are compared with one another are outlined in Section 3.10.

In this thesis, all data processing and modelling was conducted using the `R` programming language [76]. In particular, data extraction and processing used the packages `GREENGridData` [77], `dplyr` [78] and `data.table` [79]. Time series manipulation and analysis used packages `lubridate` [80], `forecast` [81], and `xts` [82]. Plots were created using the packages `ggplot2` [83], `ggplotmisc` [84], and `gridExtra` [85]. Tables were created using `knitr` [86], `pander` [87], and `kableExtra` [88]. Models were constructed using the packages `forecast` [81], `stats` [76], and `e107` [89]. Additional functions and packages used to create the models are presented after the description of the corresponding model.

To facilitate reproducibility of results for future research, significant consideration has been given to documenting each step of the analysis process. All code is publicly available under an Apache License 2.0 from `https://github.com/raffertyparker/HWCanalysis`.

3.1 Data description

The data used for this thesis was collected from monitored sub-metered electrical power usage of 44 households in Hawkes Bay and Taranaki, New Zealand, at one minute intervals between 2014 and 2018 as part of the GREEN Grid project [3]. The dataset is publicly available from the UK Data Service. Publications to date that have utilised the dataset include [90], [91], [4], [92], and [28]. In this dataset, individual households have labels beginning with `rf_`, followed by unique identifying numbers. Note that these numbers are non-sequential. This labelling format is maintained throughout this research in order to simplify cross-referencing with other research that uses this dataset. More information about this dataset, including detailed reports of data issues and access instructions, is available at `https://cfsotago.github.io/GREENGridData/`.

3.2 Initial data cleaning

Some of the households in the original dataset are not suited to the purposes of this analysis. Three were removed immediately (`rf_15`, `rf_17` and `rf_46`) due to issues with the data collection process (see `https://github.com/CfSOtago/GREENGridData/issues/21` and `https://github.com/CfSOtago/GREENGridData/issues/19` for more information). Data files from the remaining households are unzipped and processed using the `GREENGridData` package [77]. Total electricity is imputed from the submeters using the script `imputeTotalPower.R` (obtained from the `GREENGridData` Github repository). From this output, imputed total electricity demand and hot water electricity demand were extracted using `GREENGridData::extractCircuitFromCleanGridSpy1min.R`. The outputs from this script then require some further cleaning and processing to be suitable for our analysis. During the preliminary data exploration, a number of households in the dataset were found to have characteristics that meant they were unsuitable for this analysis, and were removed. These are as follows: Households `rf_07`, `rf_09`, `rf_10`, `rf_17b`, `rf_19`, `rf_21`, `rf_26`, `rf_28`, `rf_41`, `rf_43`, `rf_47` did not have separate hot water metering. Households `rf_23` and `rf_24` had hot water controlled by either a timer or a home energy management system in order to maximise self-consumption of their solar PV. Household `rf_11` had a heat-pump hot water system, which did not have a typical on/off element. Household `rf_17a` had extremely low hot water electricity values, indicating a problem with the sensor. Households `rf_27`, `rf_01`, `rf_15b`, had periods of days, weeks, or even months where no hot water electricity was used interspersed with (somewhat) normal usage. All these households were therefore discarded from further analysis. In addition, household `rf_31` only collected zero values for hot water electricity after 26th of February 2016. Rather than discarding this household, it was instead cropped so as to only contain values before this date. All remaining households are hereafter referred to as the ‘sample’ households. For the sample households, hot water electricity demand is subtracted from total electricity demand, giving two separate columns: hot water electricity, and all other electricity.

Many of the time series analysis packages used in this work require perfectly sequential data collection, with no missing (or ‘`NA`’) values. As such, any ‘holes’ in our data were dealt with as follows. When long periods of missing data occurred (determined by visual inspection of preliminary plots) the largest period of uninterrupted data collection was selected for further analysis, with the remainder discarded. The effect of removing these larger holes can be seen by comparing figures 3.1 and 3.2. Smaller holes in data were dealt with by inserting zero values of electricity power where necessary. Zero values were selected as opposed to using averages or other methods to reflect the ‘on/off’ nature of the HWC element at 1 minute timescales. This technique facilitates further analysis at this level of granularity if required.

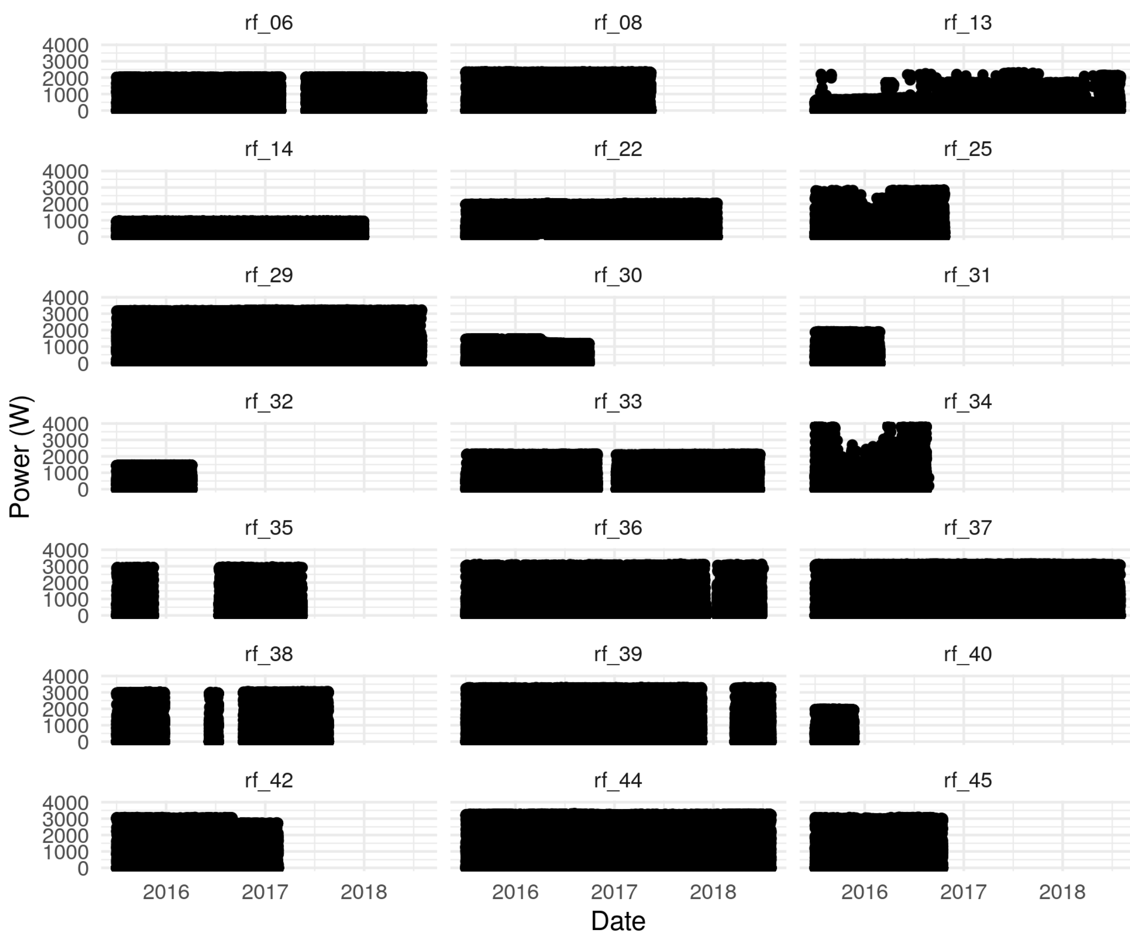

Figure 3.1: Overview of data before cleaning process

Figure 3.2: Overview of data after cleaning process

While analysis at 1 minute timescales may be beneficial in a future where metering and transmitting infrastructure is capable of facilitating control at this detail, in this thesis we are attempting to develop methods that could be implemented using existing smart meters. Smart meters in New Zealand currently store and transmit data that has been averaged over half hour periods. Before developing forecasting models we further process our data to imitate this by averaging electricity power over each half hour time step. This has the effect of 'smoothing' our data, which may be seen in Fig. 3.3.

Table 3.1: Example of the clean and processed data used in the analysis

hHour	linkID	nonHWelec	HWelec
2015-07-01 12:00:00	rf_06	497.5623	1636.8797
2015-07-01 12:30:00	rf_06	1247.1337	0.0000
2015-07-01 13:00:00	rf_06	859.3010	0.0000
2015-07-01 13:30:00	rf_06	1308.8877	1391.1207
2015-07-01 14:00:00	rf_06	668.4983	471.1483
2015-07-01 14:30:00	rf_06	321.6237	0.0000

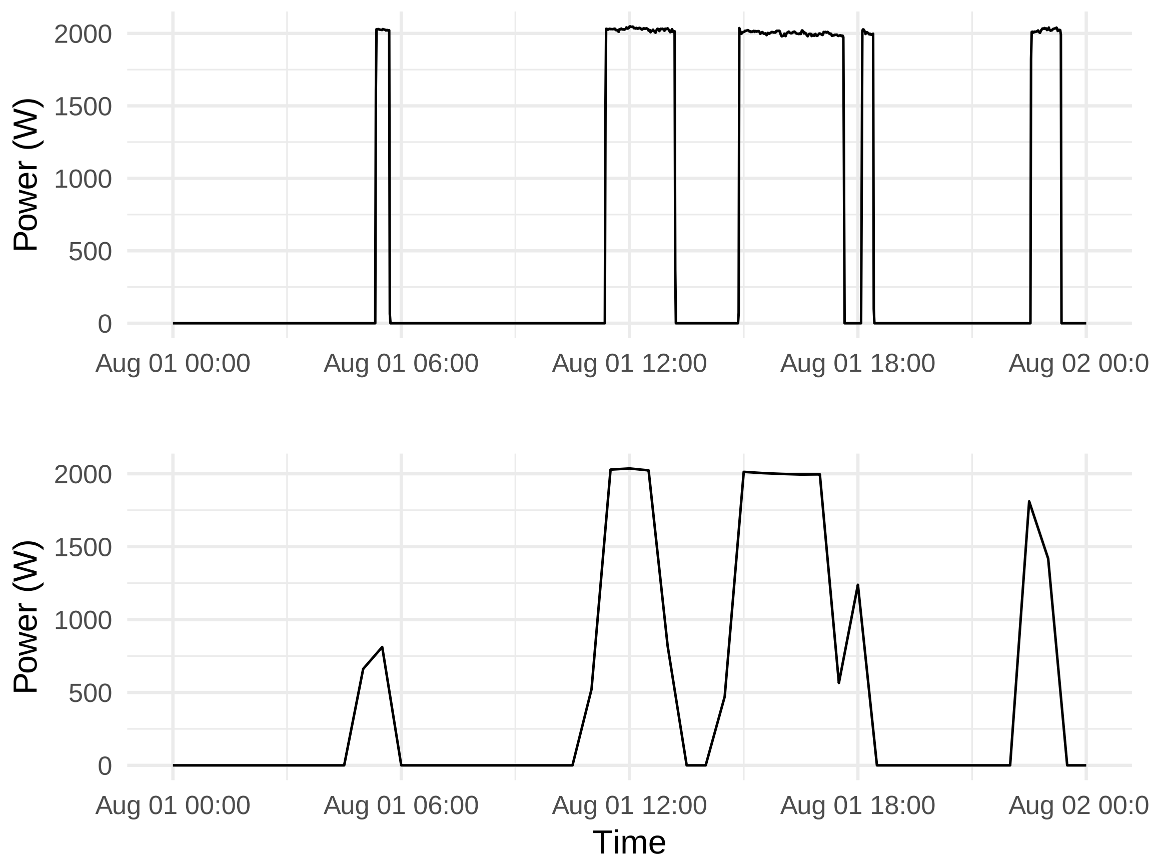

Figure 3.3: Comparison of hot water electricity data at 1 minute resolution with the same data averaged to 30 minute resolution

Note that much of the preliminary data analysis (described in Section 3.3 and presented in Chapter 4) was carried out using data at one minute resolution. All forecasting models (introduced in Section 3.7 and presented in Chapter 5) were constructed using the half hour averaged data.

While minor additional processing was necessary for particular models, the fundamental form of the data used throughout the analysis is the form shown in Table 3.1. The `hHour` column is the timestamp, `linkID` represents the household, and `nonHWelec` and `HWelec` are the half-hour averaged electricity demand of other appliances and hot water, respectively.

3.3 Time series notation and fundamentals

In this thesis we are concerned with predictability and patterns within our data over time, therefore we extensively utilise a data science technique called 'time series analysis'. This section introduces the notation and some key concepts relating to time series data and time series analysis used in this thesis. For further information or clarification, the book "Introductory Time Series With R" [33] provides a good overview of time series analysis.

A time series $\{x_1, x_2, ..., x_n\}$, also abbreviated to $\{x_t\}$, is a collection of n samples of data taken over evenly spaced intervals at discrete times $\{t = 1, 2, ..., n\}$ [33]. In this research, $\{x_t\}$ refers to the half-hour averaged hot water electricity demand. Similarly, the electricity demand of other appliances is denoted $\{y_t\}$. Models may be constructed by adjusting their internal parameters in order to best fit this data (referred to as model 'fitting'). This provides the ability to predict a future value based on historical data values. In this thesis, models are denoted using the 'hat' notation, where $\{\hat{x}_t\}$ is the *model* of $\{x_t\}$.

There are a number of terms and concepts that assist in providing clear and succinct mathematical descriptions of time series analysis methods and models. The expected value, E is often encountered in time series methods. The expected value of a discrete random variable is the probability-weighted average of all its possible values. This is defined mathematically as follows. Let x be a variable with a finite number of outcomes $x_1, x_2, \ldots, x_k$ occurring with probabilities $p_1, p_2, \ldots, p_k$, respectively. Then

$$\mathrm{E}[x] = \sum_{i=1}^{k} x_i\, p_i = x_1 p_1 + x_2 p_2 + \cdots + x_k p_k. \tag{3.1}$$

Note that when $p_1 = p_2 = \cdots = p_k$, the expected value is equal to the mean. In this thesis, the mean is denoted using overline notation, i.e., the mean of $\{x_t\}$ is $\bar{x}$.

A common statistical method used to measure the amount of variation within data is the standard deviation, σ. For a collection of T measurements of time series data, σ is given by

$$\sigma = \sqrt{\frac{1}{T-1}\sum_{i=1}^{T}(x_i - \bar{x})^2}. \tag{3.2}$$

Another useful concept is that of covariance. Covariance (denoted Cov) is a measure that is used when we have multivariable time series data (such as that comprised of both hot water electricity demand and other appliance demand). It is defined as the expected value of the product of their deviations from their individual expected values. Given two variables x and y,

$$\mathrm{Cov}(x, y) = \mathrm{E}\left[(x - \mathrm{E}[x])(y - \mathrm{E}[y])\right]. \tag{3.3}$$

In order to succinctly describe time series analysis methods it is useful to utilise the backshift operator. The backshift operator $\mathbf{B}$ shifts the value it operates on to the previous value in the series, i.e., $\mathbf{B}x_t = x_{t-1}$. This may be raised to arbitrary powers to shift values further in time, i.e.,

$$\mathbf{B}^k x_t = x_{t-k}. \tag{3.4}$$

The difference between an observed value and the value predicted by a model, $\hat{x}_t - x_t$, is formally known as the residual. This can be intuitively understood as the error in the model prediction. The residual sum of squares (RSS) is the sum of the squares of all the residuals over the times considered, i.e.,

$$\text{RSS} = \sum_{i=1}^{n} (x_i - \hat{x}_t)^2 \tag{3.5}$$

Many models considered in this research use algorithms to adjust their parameters in order to minimise the RSS subject to parameter constraints.

3.3.1 White noise

A white noise time series, $\{w_t : t = 1, 2, ..., n\}$ is a set of independent and identically distributed variables with zero mean. This implies that $\text{E}\{w_j\} = 0$ for all j, and that

$$\text{Cov}(w_k, w_j) = \begin{cases} \sigma^2, & \text{if } k = j \\ 0, & \text{if } k \neq j \end{cases} \tag{3.6}$$

where σ is the standard deviation. Models that effectively capture the underlying properties of their data have residuals that approximate white noise. This is elaborated on in Section 3.9.

3.4 Periodicity

To fit with known properties of total residential electricity demand [92], we would expect residential hot water electricity demand to fluctuate with daily and weekly periodicity due to the routines of the household occupants. This periodicity is often observed in annual timescales in econometric and financial time series data, such as an increase in house sales during summer months [93]. As many time series methods were developed to analyse econometric and financial data, these periodic fluctuations (regardless of timescale) are collectively referred to as ‘seasonality’.

3.4.1 Autocovariance

There are a number of different methods which allow us to explore seasonality within a time series. One method is autocovariance. Autocovariance compares the covariance of a stochastic process (such as our time series data of hot water electricity use) with itself at different time lags. This is a valuable tool for visualising and quantifying cyclical behaviour of data, and is defined as

$$\zeta_k = \text{E}\left[(x_t - \mu)(x_{t+k} - \mu)\right], \tag{3.7}$$

where k is the lag value, E is the expected value operator, and μ is the mean of x_t and x_{t+k}, i.e. $\mu = \frac{x_t + x_{t+k}}{2}$.

Autocovariance is visualised through plotting the autocorrelation function. A lag k autocorrelation function ρ_k is defined by

$$\rho_k = \frac{\zeta_k}{\sigma^2}. \tag{3.8}$$

The autocorrelation function ρ_k is then plotted against lag values k. For our analysis, this was obtained using the `Acf` function [81].

3.4.2 Seasonal and trend decomposition

Another time series method that may be used to explore seasonality of time series data is by approximating the data as a function or combination of functions. This facilitates the discovery of underlying patterns within the data. Local polynomial regression (Loess) is a method that fits simple polynomial functions to small localised subsets of data by minimising the function's RSS.

Seasonal and trend decomposition using Loess (STL) is a procedure used for discovering underlying patterns in the data. An STL decomposition separates the data into three components, referred to as the 'trend', 'seasonality', and 'remainder' (also referred to as 'random'). The original data can be replicated by summation of these three components.

The trend represents low frequency changes in the data, along with longer term average shifts. Seasonality refers to the periodic behaviour of the data. The remainder is the deviation of the actual data from the addition of the trend and the seasonality.

The STL algorithm requires two recursive procedures, an inner loop nested within an outer loop, both of which are lengthy and involved. Its description is therefore beyond the scope of this thesis, but can be examined in [94]. For our analysis, STL decomposition was conducted using the `R` packages `stats` [76] and `forecast` [81]. Decomposition of data using STL can be a highly effective addition to a hot water demand model, as documented in [26]. Within this research, we combine an STL model with variations of an ARIMA model, as outlined in Sections 3.7.9 and 3.7.11.

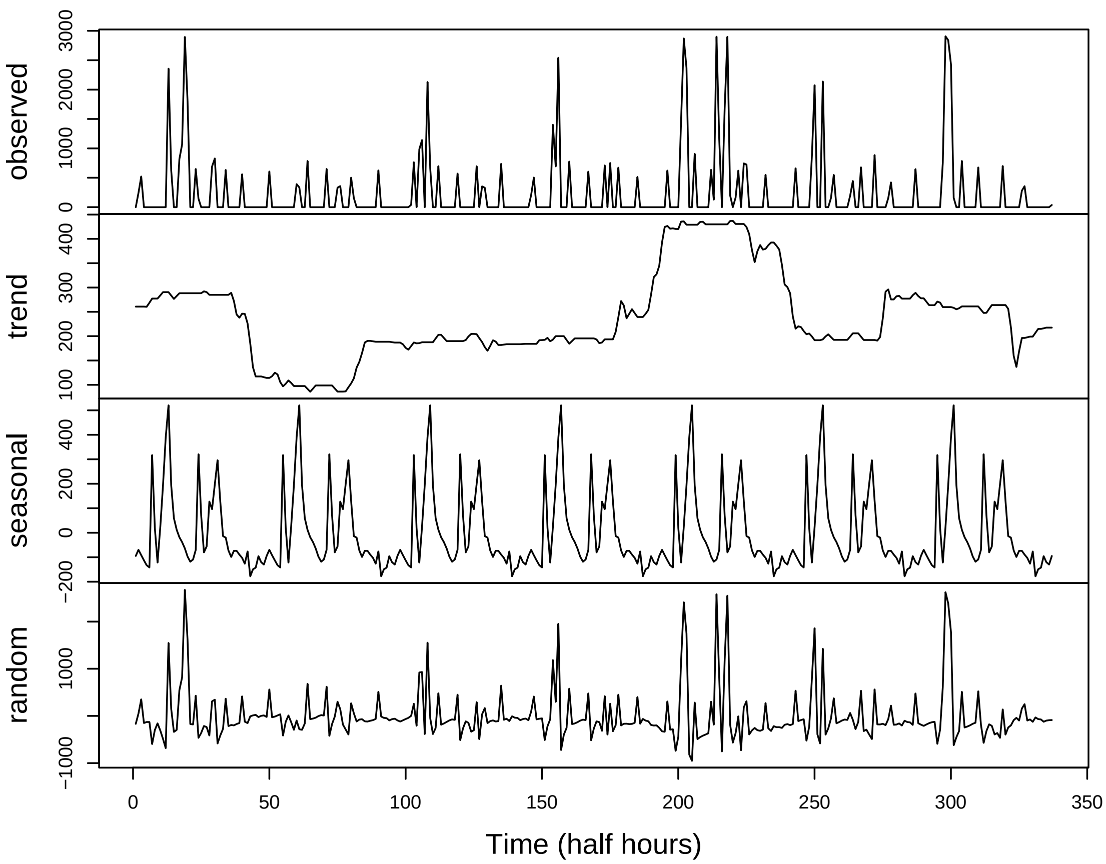

Figure 3.4: Seasonal decomposition of one week of hot water electricity demand data (W) from household rf_35, performed by the `decompose` function in the `stats` package [76]

Fig. 3.4 illustrates an STL decomposition of a household over one week. The top panel, labelled 'observed', is the actual data, while the lower three panels display the trend, seasonal, and random components of this data. Note the repeating daily pattern in the seasonal panel.

While inspecting a plot of the STL decomposition provides a good visual intuition of patterns in the data, the model is not directly interpretable in its raw form. Each component is provided as a column of values spanning over the same time steps as the data to which it is fitted. The seasonal component can be obtained as a daily profile provided by its 48 repeating half-hour values. Succinct parameters for the trend component can be obtained through fitting a polynomial to it and taking the coefficients. This additional processing of the STL decomposition is left as further work.

3.4.3 Fourier analysis

Fourier analysis is another method of approximating time series data in terms of functions in order to discover underlying properties of the data. Rather than fitting polynomial functions to local subsets of data as performed by STL, a Fourier analysis represents data in terms of a linear combination of sinusoids, known as a Fourier series [95].

When applied to a collection of n observations, $\{x_1, x_2, \ldots, x_n\}$ a Fourier series may be described mathematically as

$$\hat{x}_t = c + \sum_{k=1}^{[n/2]} \{a_k \cos(\omega_k t) + b_k sin(\omega_k t)\}, , \tag{3.9}$$

where c is a constant, ω_k are the frequencies, and a_k, b_k are the amplitudes [96]. These parameters are adjusted in order to best fit the equation to the underlying data. Dominant frequencies are those which correspond (i.e., have the same value of k) to the sinusoids with the highest amplitudes [97]. The mechanics of how the parameters of (3.9) are adjusted to best fit the data are beyond the scope of this thesis. For more information, refer to [97] or [96].

As some time-series models (such as seasonal ARIMA) require cycle frequency to be manually input, automated frequency extraction has the potential to be highly beneficial when creating household-specific prediction models. To provide an example related to this research, some households may display weekly seasonality, others may display stronger seasonality in daily, half-daily, or other timescales. Rather than using a 'one size fits all' seasonality, or manually determining the seasonality through autocovariance, Fourier analysis may provide a means of automating a household-specific model building process.

Within this research, dominant cycle frequencies (seasonal periods) within our data are extracted by Fourier analysis using the function `periodogram` from the `TSA` package. These were intended to be used as inputs for a seasonal ARIMA model (refer to Sections 3.7.8 and 5.5), however this model was eventually discarded as its computational time was prohibitively long.

3.5 Stationarity

A stationary time series is one whose joint probability distributions are stable over time [93]. This means that for a time series $\{x_t\}$, any sequential subset of the data should have the same expected value and variance (σ^2) as any other sequential subset, i.e.

$$E[x_{t_1}, \ldots, x_{t_n}] = E[x_{t_1+\tau}, \ldots, x_{t_n+\tau}] \tag{3.10}$$

and

$$\sigma^2[x_{t_1}, \ldots, x_{t_n}] = \sigma^2[x_{t_1+\tau}, \ldots, x_{t_n+\tau}], \tag{3.11}$$

for all integer values of τ, n such that $x_{t_n}, x_{t_n+\tau}$ are within $\{x_t\}$.

Stationary time series data should not show signs of trends or seasonality. Non stationary data may be made stationary by a process known as differencing (see Section 3.7.4). For an example of non-stationary data made stationary through differencing, see Fig. 3.6.

Some models utilised in this research (in particular those that are ARIMA based) are more accurate when applied to stationary data. If non-stationarity is detected, ARIMA models are fitted to differenced data to achieve an accurate fit, and then 'un-differenced' before returning an output. Tests of data stationarity are obtained automatically within the `auto.arima` [81] function. Details of how this is carried out can be found in the documentation of [81].

3.6 Correlation with other appliance electricity demand

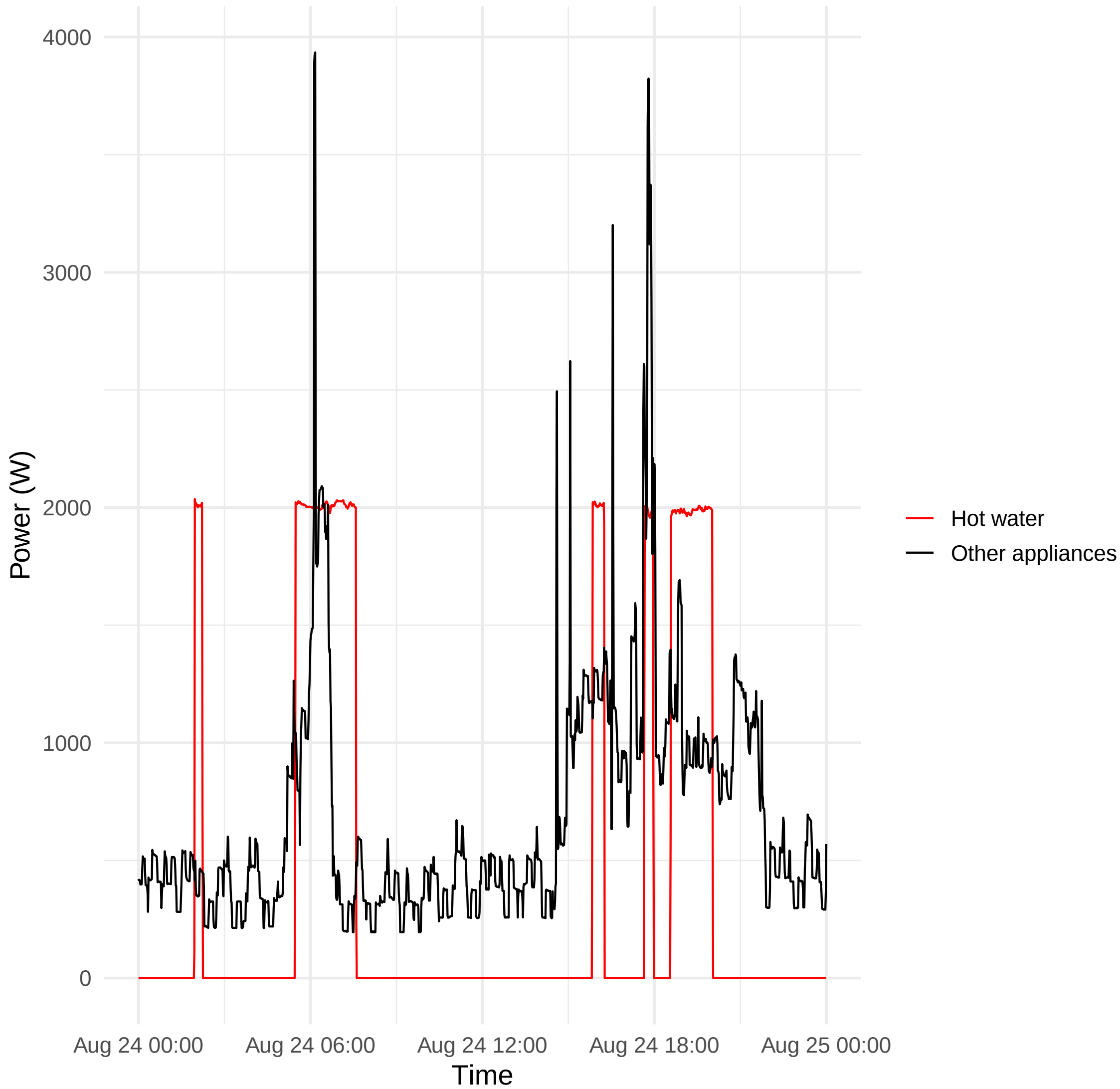

Figure 3.5: Electricity demand of hot water and other appliances over one day for household `rf_06` at 1 minute resolution

During initial data exploration, it was found that households often show instances whereby the electricity demand of other appliances is correlated with hot water electricity demand. In general this correlation is unsurprising, as it simply confirms that the houses tend to be occupied at the time that hot water is being used. However, this correlation becomes useful for forecasting when values of hot water demand are correlated with prior values of other electricity demand. Many people may choose, for example, to turn on the kettle before a shower in the morning, or cook dinner before doing the dishes in the evening. An example demonstrating this behaviour is given in

Fig. 3.5. We can make a more thorough exploration into the temporal relationship between hot water electricity demand and that of other appliances through examining their cross-covariance.

In a similar fashion to the autocovariance function given by (3.7), the cross-covariance function of two variables x, y is defined by

$$\gamma_k(x,y) = \mathrm{E}\left[(x_{t+k} - \mu_x)(y_t - \mu_y)\right], \tag{3.12}$$

where variable x lags variable y by lag k, $\mu_x = \frac{x_t + x_{t+k}}{2}$, and $\mu_y = \frac{y_t + y_{t+k}}{2}$ [33].

Plotting the cross-covariance (known as a *cross correlogram*) allows visual inspection of the relation between the two variables at different time lags. Cross-covariances were constructed using the `Ccf` [81] function. When creating models that incorporate regression of lagging values of other electricity demand (refer to Sections 3.7.3, 3.7.10, 3.7.11 and 3.7.12), cross-covariances allow us to select appropriate lag times.

3.7 Introduction to selected models

This section outlines a broad introduction to the various models selected for use in this thesis. These models were selected based on their performance in similar applications, with consideration given to the aims of this thesis outlined in Section 1.5. Due to the range of metrics by which models are compared (see 3.10), some relatively simplistic models were included for comparison. While these simple models may not perform as well as more complex ones in terms of accuracy, they have the potential to perform higher in other metrics. Models are presented roughly in order of their complexity.

Note that white noise terms, $\{w_t\}$ are included within model descriptions. This term represents the residual of the model, as discussed in Section 3.3. Previous values of these white noise residuals are used to predict future values of the dependent variable in moving average models. As described in Section 3.3, a white noise time series is inherently unpredictable with an expected value of zero. Thus, when a model is used for forecasting, predictions of future values of these terms are always equal to zero. If the same model was used for demand simulations, non-zero white noise terms can be artificially generated to provide realistic fluctuations of hot water demand about the values predicted by the model [25].

3.7.1 Naive model

In keeping with comparative forecasting research best practice [26], [66], a very simplistic model is selected by which to compare the performance of more complicated models to. The model selected for this task is known as a 'random walk'. A random walk model takes the form

$$\hat{x}_t = x_{t-1} + w_t, \tag{3.13}$$

where $\{w_t\}$ is a white noise series.

Random walk models are more commonly used in simulations than forecasts. When used in simulations, the white noise is artificially generated to simulate a potential evolution of the variable in time. When used in forecasting, as always within this thesis, the white noise term is allocated its expected value of zero. Consequently, the value of hot water electricity demand x of the next time step can be best predicted by its value in the current time step, i.e. $\hat{x}_{t+1} = x_t$. Due to their simplicity and ease of computation random walk models are sometimes used in forecasting as a benchmark model, against which other models may be compared. For this reason, they are often referred to as 'naive' models. The naive models used in this research were created using the `naive` function [81]. Further details regarding this model can be found in [33].

3.7.2 Seasonal naive model

In a similar manner to the naive model, the seasonal naive model estimates the next value in a series from a single prior observation. However, the seasonal naive model makes the assumption that for data that displays seasonality, the most likely value of the next time step is that of the same time one period prior, i.e.

$$\hat{x}_t = x_{t-s} + w_t, \tag{3.14}$$

where s is the length of one period. Seasonal naive models were created using the `snaive` function [81]. Further details regarding this model can be found in [33].

3.7.3 Simple linear regression

Linear modelling is a regression method prized for its simplicity in interpretation and computation. A special case of a linear model is the simple linear regression, which fits a straight line through data in order to minimise the RSS. When applied to forecasting, a simple linear regression can fit a line to historical values of the dependent (or 'response') variable in order to estimate future values. This line is fitted to minimise the RSS, and is given by $x_t = \gamma_0 + \gamma_1 t$. While this can be useful for determining general trends over longer periods of time, it can not capture any regular shorter term fluctuations of a time series about this trend (seasonality).

Another way of utilising simple linear regression for time series forecasting is by introducing a separate predictor variable or variables. For our data, we separate electricity demand into hot water, and other appliances. Simple linear regression then provides a method of exploring how the demand of hot water electricity (x_t) can be predicted from previous values of other appliances (y_{t-k}). This is given by the model

$$\hat{x}_t = \gamma_0 + \sum_{i=1}^{k} \gamma_i y_{t-i} + w_t. \tag{3.15}$$

In this thesis, simple linear regression models were constructed using the `lm` function [76]. Further details regarding simple linear regression can be found in [33].

3.7.4 Differencing

Differencing a time series $\{x_t\}$ is a simple method of removing trends in order to make the time series stationary (see Section 3.5). This is often used in combination with other methods which become more accurate when applied to stationary data. Differencing is defined using the backshift operator as

$$I^d = (1 - \mathbf{B})^d, \tag{3.16}$$

where d is an integer. A time series $\{x_t\}$ is referred to as *integrated* of order d if the dth difference of $\{x_t\}$ is stationary. Integrated time series' become relevant when considering the ARIMA model in Section 3.7.7, as this is the component within the ARIMA model that the 'I' refers to. An example of first order integrated time series data is demonstrated in Fig. 3.6, whereby total household electricity data becomes stationary after it is differenced.

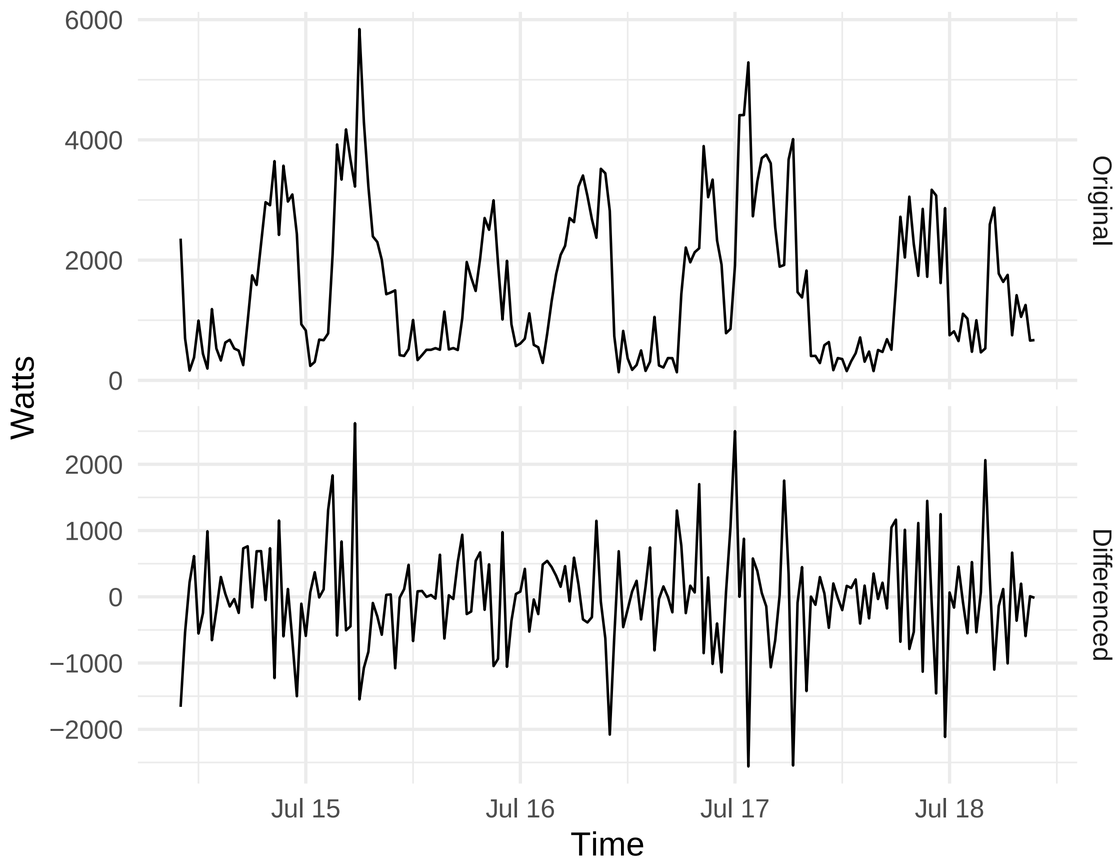

Figure 3.6: Total electricity demand data of household `rf_22` before and after first-order differencing

3.7.5 Autoregression

An autoregression model of order p, referred to as $AR(p)$, can be given by

$$\hat{x}_t = \alpha_1 x_{t-1} + \alpha_2 x_{t-2} + ... + \alpha_p x_{t-p} + w_t, \tag{3.17}$$

where $\{w_t\}$ is white noise and α_i are model parameters, and $\alpha_p \neq 0$. This can be represented using the backshift operator as

$$\hat{x}_t = \sum_{i=1}^{p} \alpha_i \mathbf{B}^i x_t + w_t, \tag{3.18}$$

and by moving the summation to the left hand side, this may be expressed in polynomial notation as

$$\theta_p(\mathbf{B})\hat{x}_t = (1 - \alpha_1\mathbf{B} - \alpha_2\mathbf{B}^2 - ... - \alpha_p\mathbf{B}^p)\hat{x}_t = w_t, \tag{3.19}$$

where θ_p is a polynomial of order p. Autoregression makes up the 'AR' component in an ARIMA model, discussed further in Section 3.7.7.

3.7.6 Moving average

A q-order moving average model, $MA(q)$ can be expressed as a linear combination of the white noise residual w_t (see Section 3.3) and the q most recent previous residuals, defined as

$$\hat{x}_t = \mathrm{E}[x_t] + w_t + \beta_1 w_{t-1} + ... + \beta_q w_{t-q}. \tag{3.20}$$

If $\mathrm{E}[x_t] = 0$, which may be artificially induced by prior processing with the differencing method in the Section 3.7.4, (3.20) may be expressed as

$$\hat{x}_t = (1 + \beta_1\mathbf{B} + \beta_2\mathbf{B}^2 + ... + \beta_q\mathbf{B}^q)w_t = \phi_q\mathbf{B}w_t, \tag{3.21}$$

where $\mathbf{B}$ is the backshift operator defined in (3.4), and ϕ_q is a polynomial of order q. A moving average model makes up the 'MA' component in an ARIMA model, discussed further in Section 3.7.7.

3.7.7 ARIMA

An Autoregressive Moving Average (ARMA) process of order (p, q) combines autoregression with the moving average process, adding the two together. This results in

$$x_t = \alpha_1 x_{t-1} + \alpha_2 x_{t-2} + ... + \alpha_p x_{t-p} + w_t + \beta_1 w_{t-1} + \beta_2 w_{t-2} + ... + \beta_q w_{t-q}. \tag{3.22}$$

This may be expressed in terms of the backward shift operator in polynomial form as

$$\theta_p(\mathbf{B})x_t = \phi_q(\mathbf{B})w_t. \tag{3.23}$$

An ARMA model can make predictions about future values based on previous values. In the context of hot water electricity demand forecasting, the ARMA model would recognise that the element used certain values of electricity over the previous few time steps, and provides a value for the next time step accordingly.

For reasons that are beyond the scope of this thesis, ARMA models are more accurate when applied to stationary data. Thus to improve accuracy, they are often applied to data that has been integrated in order to force stationarity. If data is integrated d times before an ARMA(p, q) model is fitted, the output is referred to as an ARIMA(p, d, q) model. Autoregression and moving average models may then be considered individually as special cases of ARIMA models; ARIMA$(p, d, 0)$ and ARIMA$(0, d, q)$ respectively. In the context of hot water electricity demand forecasting, the ARIMA model would recognise that, given the subsequent changes in element electricity use over the previous few time steps, a prediction can be made as to the following change in electricity use.

When fitting an ARIMA model, values for p, d and q must be selected in order to best fit the data while minimising computational expense and avoiding overfitting. Larger values of p and q in particular tend to increase accuracy, while taking longer to compute. The process of selecting optimal values for p, d and q can be automated through minimising the Akaike Information Criterion (AIC) [98], where

$$AIC = -2 \times \text{log-likelihood} + 2 \times \text{number of parameters}. \tag{3.24}$$

The `R` function `auto.arima` [81] was used to create ARIMA models. This function automatically select the parameters p, d, q which minimise the AIC specific to the particular data being modelled. This is done iteratively according to the algorithm outlined in [99]. Inputs for maximum values of p, d and q are necessary in order to bound processing time. These maximum values were (respectively) fixed at 5, 2 and 5 for all ARIMA based modelling conducted within this research. Further details regarding ARIMA models can be found in [33].

3.7.8 Seasonal ARIMA

In a similar manner to how trends can be removed through differencing at lag 1, seasonal effects within data can be removed by differencing at lag s, where s is the length of the season. A seasonal ARIMA model may also introduce additional autoregressive and moving average terms at lag s, giving a model of the form $ARIMA(p, d, q)(P, D, Q)_s$. This may be expressed in polynomial notation as

$$\Theta_P(\mathbf{B}^s)\theta_p(\mathbf{B})(1 - \mathbf{B}^s)^D(1 - \mathbf{B})^d x_t = \Phi_Q(\mathbf{B}^s)\phi_q(\mathbf{B})w_t. \tag{3.25}$$

Due to residential hot water demand displaying seasonality (refer to Section 4.2), seasonal ARIMA models were a promising candidate for the aims of this thesis.

3.7.9 STL with ARIMA

An alternative mechanism by which to incorporate cyclic effects into ARIMA models is through applying STL decomposition (see Section 3.4.2) before model fitting. A time series may be split into seasonal, trend, and remainder components, with the remainder component being modelled as an ARIMA process in the same manner as described in Section 3.7.7. The seasonal and trend components are then added back to the ARIMA modelled remainder as the complete forecasting model. This method was the most accurate model considered in [26]. STL + ARIMA models were fitted using the `stlm` function [81].

3.7.10 ARIMAX

An 'ARIMAX' model is an ARIMA model with the addition of an external regressor [66]. One way in which this may be interpreted is as a simple linear regression model with ARIMA errors. This combines Equations (3.15) and (3.22) in the form

$$\hat{x}_t = \gamma_0 + \sum_{i=1}^{k} \gamma_i y_{t-i} + \sum_{i=1}^{p} \alpha_i x_{t-i} + \sum_{i=1}^{q} \beta_i w_{t-i} + w_t, \tag{3.26}$$

where y is the external regressor. In the context of this research, the regressor y is the electricity demand of other appliances. ARIMAX models were created using the `auto.arima` function [81].

3.7.11 STL with ARIMAX

The predictive power of seasonal decomposition and external regressors may be combined with an ARIMA model, to get a model we refer to as STL + ARIMAX. This method decomposes the data into seasonal, trend and remainder components, (refer to Section 3.4.2), and then fits an ARIMAX model (refer to Section 3.7.10) to the remainder component using lagged values of other appliance electricity demand as external regressors (refer to Section 3.7.10). This is then added back to the seasonal and trend components to complete the model. STL + ARIMAX models were created using the `stlm` function [81]. No existing literature was discovered that uses an STL + ARIMAX model for electricity demand forecasting.

3.7.12 Support vector machines

Support vector machines (SVMs) are an AI method commonly used for forecasting electricity demand. This process allows data that may highly non-linear to be a linearly classified by mapping it in a higher dimensional space [100]. To facilitate intuition of this process, an artificial example of this is provided in Fig. 3.7. Fig. 3.7 shows how, by transforming data from a two dimensional space into a three dimensional space using an appropriate function, the data may be linearly separated, in this case by a plane.

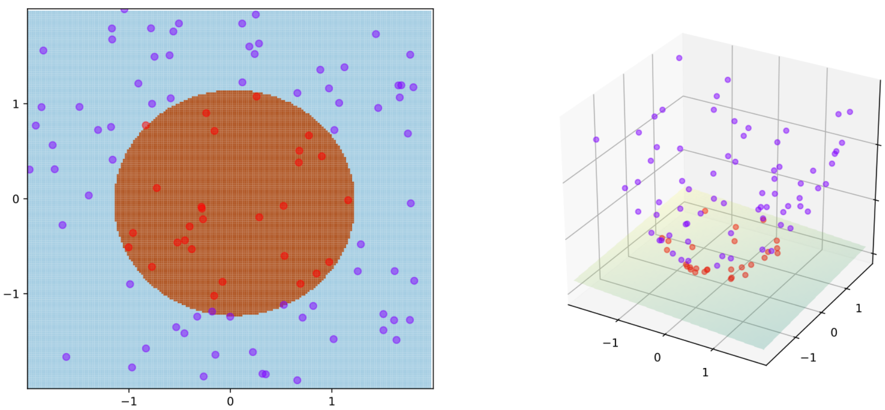

Figure 3.7: Example demonstrating the underlying classifying mechanism of a SVM, whereby data with two different classifications (signified by colour) are mapped into a higher dimension to permit a linear separation [101]

The function use to perform this mapping is known as the kernel function, K, In Fig. 3.7, $K(x_1, x_2) = (x_1, x_2, x_1^2 + x_2^2)$. This technique may be achieved in arbitrarily many dimensions, with the high-dimensional plane used to separate the data known as a hyperplane. The position of the classifying hyperplane is constructed iteratively in order to maximise the perpendicular distance between the hyperplane and the closest samples on either side of it. For this reason, the classifying hyperplane is denoted the 'maximum margin' hyperplane. The vectors that run parallel to the hyperplane and contain the closest samples to it are known as 'support vectors'.

The SVMs used in this research were built in the following manner. First, we denote the maximum margin hyperplane as Γ, which is defined mathematically in terms of the variables within our data as

$$\Gamma = \gamma_0 h_t + \sum_{i=1}^{k} \gamma_i x_{t-i} + \sum_{i=1}^{j} \gamma_{k+i} y_{t-i}, \tag{3.27}$$

where h is the (half) hour of day, provided to take seasonality into consideration, x and y again denote hot water electricity and the electricity demand of other appliances respectively, and γ refer to unknown weights that must be determined through the learning algorithm. Now Γ is fitted iteratively in the following manner.

For simplification, we denote $\mathbf{z}_t$ to be the vector comprised of our data variables, $\mathbf{z}_t := \{h_t, x_{t-1}, \dots x_{t-k}, y_{t-1}, \dots y_{t-k}\}$. Now (3.27) can be expressed as

$$\Gamma = b + \sum \alpha_i \Gamma_i K(\mathbf{z}_t(i), \mathbf{z}_t). \tag{3.28}$$

In the context of iteratively fitting Γ, vector $\mathbf{z}_t$ can be thought of as the most recent data sample provided to the fitting algorithm. Expressed in this manner, Γ is defined by parameters b and α_i, and $\mathbf{z}_t(i)$ are the support vectors. Γ_i is called the 'class value' of $\mathbf{z}(i)$, and only takes two values, 1 or -1. This can be understood intuitively as classifying $\mathbf{z}_t$ according to whether it sits 'above' or 'below' Γ. A Gaussian radial basis function was selected according to preliminary modelling as the optimal kernel function, K. This is defined as

$$K = e^{(-|\mathbf{z}_t(i)-\mathbf{z}_t|^2)}. \tag{3.29}$$

Parameters b and α_i in (3.28) are adjusted by solving a quadratic optimisation problem. This is outside the scope of this thesis, but may be found in [102]. Once these parameters have been optimised, the original weights γ in (3.27) can be determined. Following this, predictions for our hot water value x_t can be obtained by inputting corresponding $\mathbf{z}_t$ vector values of $h_t, x_{t-1}, x_{t-2}, y_{t-1}, y_{t-2}$ into (3.27). In keeping with the notation of the rest of this chapter, we may then denote the SVM model as

$$\hat{x}_t = \Gamma(\mathbf{z}_t). \tag{3.30}$$

Some of the more involved details regarding SVMs have been withheld as they are outside the scope of this thesis. For further details regarding using SVM models for forecasting electricity demand refer to [65], or for a more general overview, refer to [100]. Support vector machines were created using the `svm` function [89]. This function uses the training algorithms detailed in [102].

3.8 Training and validating

When fitting a prediction model to data, a closer fit can usually be obtained by increasing the number of parameters within the model. An extreme example of this would be a highly complex model with zero, or close to zero errors. While a cursory look at the residuals of this model might suggest it has high prediction powers, it may start to return large errors once used to predict data it has not encountered before. This is an example of 'overfitting' a model, whereby a model goes beyond capturing key statistical properties of the underlying data and begins fitting itself to the random fluctuations about these properties, which are inherently unpredictable. For this reason, it is good practice to fit a model to one set of data, and then test its accuracy by making predictions on a separate set [103]. These two separate sets are referred to as training data and validating data, respectively.

Within this research, household data were separated chronologically. For each household, the first 80% of the data were used for model training, and the final 20% were used for validating. All models within this research were built and tested in this manner, with the exception of the two naive benchmark models. As, by definition, the naive models had no chance of becoming overfit, they were simply tested against the validation data.

3.9 Residual analysis

If a model has accurately captured the underlying statistical properties of its data, the model residuals will resemble white noise. A simple diagnostic test may be used to check that this is the case. Following from the definition of a white noise series provided in Section 3.3.1, and the autocorrelation function in (3.8), a white noise series has an autocorrelation function ρ_k such that

$$\rho_k = \begin{cases} 1, & \text{if } k = 0 \\ 0, & \text{if } k \neq 0. \end{cases} \tag{3.31}$$

Due to natural variations in the data, statistically effective models will not have residuals such that ρ_k is *exactly* zero for all $k \neq 0$. Instead, the plot of ρ_k should start at one and decay rapidly below a 5% significance level (shown on the autocovariance plots in Chapter 5 as a blue dotted line). Effective models should have at most 5% of values exceeding this level [33]. Slower decay indicates that autoregressive properties have not been sufficiently considered. Periodic oscillations that exceed the significance level indicate that seasonal properties have not been sufficiently considered.

3.10 Comparative metrics

There are a number of different considerations that must be made when comparing models for the process of electricity demand forecasting. This section outlines those considered relevant to the aims of this thesis.

3.10.1 Accuracy

Perhaps the most important consideration in forecasting is the accuracy of the model in predicting values from the set of validation data. While there are a number of ways that model accuracy could be defined, a common method in existing literature is that of the 'root mean square error' (RMSE) [26], [27], [50], [54], [56], [58], [60], [61], [64], [104]. This is determined by the root mean square of the residuals, with lower values indicating higher accuracy. Expressed mathematically, the RMSE of predicted values $\hat{x}_t$, where actual values are x_t and predictions are observed over T time intervals, is given by:

$$\text{RMSE} = \sqrt{\frac{\sum_{t=1}^{T} (\hat{x}_t - x_t)^2}{T}}. \tag{3.32}$$

For each model considered within this research, the average RMSE of predictions is taken over all households to provide the overall model RMSE.

As demand response is most crucial during daily peak periods, additional analysis is carried out to ascertain the accuracy during grid peaks (from 7 am to 9 am, and from 5 pm to 8 pm [10]). This was obtained by calculating the RMSE for all predictions that occurred during peak periods (denoted $\text{RMSE}_{\text{peak}}$). To assist comparisons, this is then used to calculate a percentage error increase (PEI) between the average RMSE and the average RMSE during peak times. The PEI is given by:

$$\text{PEI} = \frac{\text{RMSE}_{\text{peak}} - \text{RMSE}}{\text{RMSE}}. \tag{3.33}$$

3.10.2 Physical fidelity

In addition to accuracy of prediction values, there are benefits to models that closely resemble the physical process they are predicting. A physically accurate model of electricity demand would be better suited to incorporating into a physical model than a model with more accurate RMSE.

As an extreme example, imagine two models, one which precisely matched the general 'shape' of the data, but was consistently wrong in predictions about when exactly the demand occurred. The other simply estimated the element to assume its mean value at all times. It is possible the latter model would have a lower RMSE than the former, however it would clearly be less valuable in demand simulations.

The models utilised in this thesis assume the dependent variables are continuous and have no upper or lower bounds. When strictly focussing on fitting to data in order to minimise RSS, a model may have a significantly different shape than the underlying data. In addition, models may make predictions that are negative or greater than the element capacity. These would not be optimal to use in physical modelling, as they are clearly physically inaccurate. An optimal electricity demand simulation should capture the physical process of the hot water cylinder element turning on and off in response to the drawdown of hot water according to residential demand patterns [28]. This is referred to as physical fidelity. Properties of decent physical fidelity include the non-existence of negative values or values above the maximum power of the element being modelled, as well as replication of the general shape of the data (determined qualitatively by visual inspection).

3.10.3 Interpretability

Another important consideration for research purposes is interpretation of results. Model interpretability is a measure of how well we can infer fundamental properties of hot water demand from the model. Models that are easy to interpret are valuable for understanding the human behaviour behind hot water use, a useful insight when building simulations. For a model to score highly in this metric, it should be composed of succinct equations, with easily obtainable parameters. 'Black box' models and those that are comprised of a large number of parameters score poorly in this metric.

3.10.4 Computational speed

In order to make the predictions necessary to effectively participate in smart control for demand response, models for hundreds of thousands of households need to be fitted. While models only need to be fitted once in order to provide predictions, changes in household occupancy and demand patterns means that these models would need to be updated on a regular basis. In addition, when researching hot water demand patterns for demand simulations there is value in being able to explore data patterns without waiting a long time for models to be constructed. For these reasons, consideration is given to the computational speed of each model. This is defined as the amount of time taken to fit the model, with all models fitted to all households sequentially using the same machine. To avoid the model fitting slowing down due to memory allocation issues, the process used in this thesis fitted all models to an individual household, and then moved onto the next household, progressing sequentially through all households. The final metric for comparison was the average time taken to fit a model to a household. When a model had an average fitting time of under 0.1s, its computational times were considered 'negligible'.

3.11 Summary

This chapter has provided mathematical details of the analysis methods and forecasting models that were used in this thesis. These included formulas for:

- autocorrelation (3.8)
- Fourier series' (3.9)
- crosscovariance (3.12)
- random walk (3.13)
- simple linear regression (3.15)
- autoregressive moving averages (3.22)
- support vector machines (Section 3.7.12)

It also described the comparative metrics by which to judge model performance, such as:

- accuracy (Section 3.10.1)
- physical fidelity (Section 3.10.2)
- interpretability (Section 3.10.3)
- computational speed (Section 3.10.4)

The following chapters, 4 and 5, provide the results obtained by these analysis methods and forecasting models.

Chapter 4

Preliminary data analysis

This chapter presents the results from the preliminary data analysis that was carried out before starting to build forecasting models. A brief summary of the data is provided along with key visualisations regarding patterns of hot water use. Next, the results of the data analysis techniques introduced in Section 3.3 are presented. These results are discussed briefly in terms of their implications for the models presented in Section 5.

4.1 Data summary

Table 4.1 shows the mean values of hot water electricity demand and other appliance electricity demand for sample households. Mean demand varies significantly for each household, ranging from 165W (household `rf_30`) to 501W (household `rf_38`). The mean value of hot water demand over all households (mean of means) is 315W.

Table 4.1: Mean values of electricity demand for hot water and other appliances for sample households at half-hourly resolution, rounded to the nearest Watt

Household	Hot water (W)	Other appliances (W)
rf_06	394	506
rf_08	276	767
rf_13	213	1233
rf_14	223	422
rf_22	391	956
rf_25	258	599
rf_29	340	1249
rf_30	165	509
rf_31	207	407
rf_32	286	398
rf_33	360	569
rf_34	308	784

Household	Hot water (W)	Other appliances (W)
rf_35	230	1090
rf_36	290	543
rf_37	280	291
rf_38	501	506
rf_39	393	1504
rf_40	356	899
rf_42	355	848
rf_44	478	598
rf_45	316	443

Table 4.2 shows the median values of hot water electricity demand and other appliance electricity demand for sample households. Median values for hot water electricity demand equal zero for most households, however some display non-zero medians. These non-zero median values may indicate very frequent hot water usage or lack of insulation around the cylinder.

Table 4.2: Median values of electricity demand for hot water and other appliances for sample households at half-hourly resolution, rounded to the nearest Watt

Household	Hot water (W)	Other appliances (W)
rf_06	0	469
rf_08	0	487
rf_13	0	922
rf_14	0	223
rf_22	205	598
rf_25	190	451
rf_29	0	1034
rf_30	0	268
rf_31	0	196
rf_32	0	244
rf_33	315	417
rf_34	0	460
rf_35	0	912
rf_36	59	235
rf_37	0	175
rf_38	0	265
rf_39	0	1144
rf_40	80	579
rf_42	0	393
rf_44	0	286
rf_45	0	192

Variation in hot water electricity demand can also be seen in the box and whisker plot shown in Fig. 4.1. The left and right hand sides of the boxes within Fig. 4.1 correspond to the first and third quartiles respectively. The bold vertical line corresponds to the second quartile. The right hand side 'whiskers' extend to 1.5 $\times$ IQR from the third quartile (where 'IQR' is the inter-quartile range, or distance between the first and third quartiles). The mean is included in red. Data beyond the end of the whiskers are considered "outliers" and are plotted individually [83]. Due to the large number of zero values within the data, the inter-quartile range within each household is highly restricted with respect to the total range. Points to the right hand side of each whisker (which are numerous enough to appear as a bold line) would normally be considered outliers by this analysis method. However, it is important that the models constructed in this thesis are able to predict these values, as they indicate times where the element is reheating the water after a significant usage event. This plot also gives an indication as to the maximum power output for each household. Assuming there was at least one instance during the year where the element was on for a full half hour, the far right hand side point of each household should correspond approximately with its maximum power output. Examination of Fig. 4.1 indicates that most households have an approximate maximum power output of either 2 kW or 3 kW.

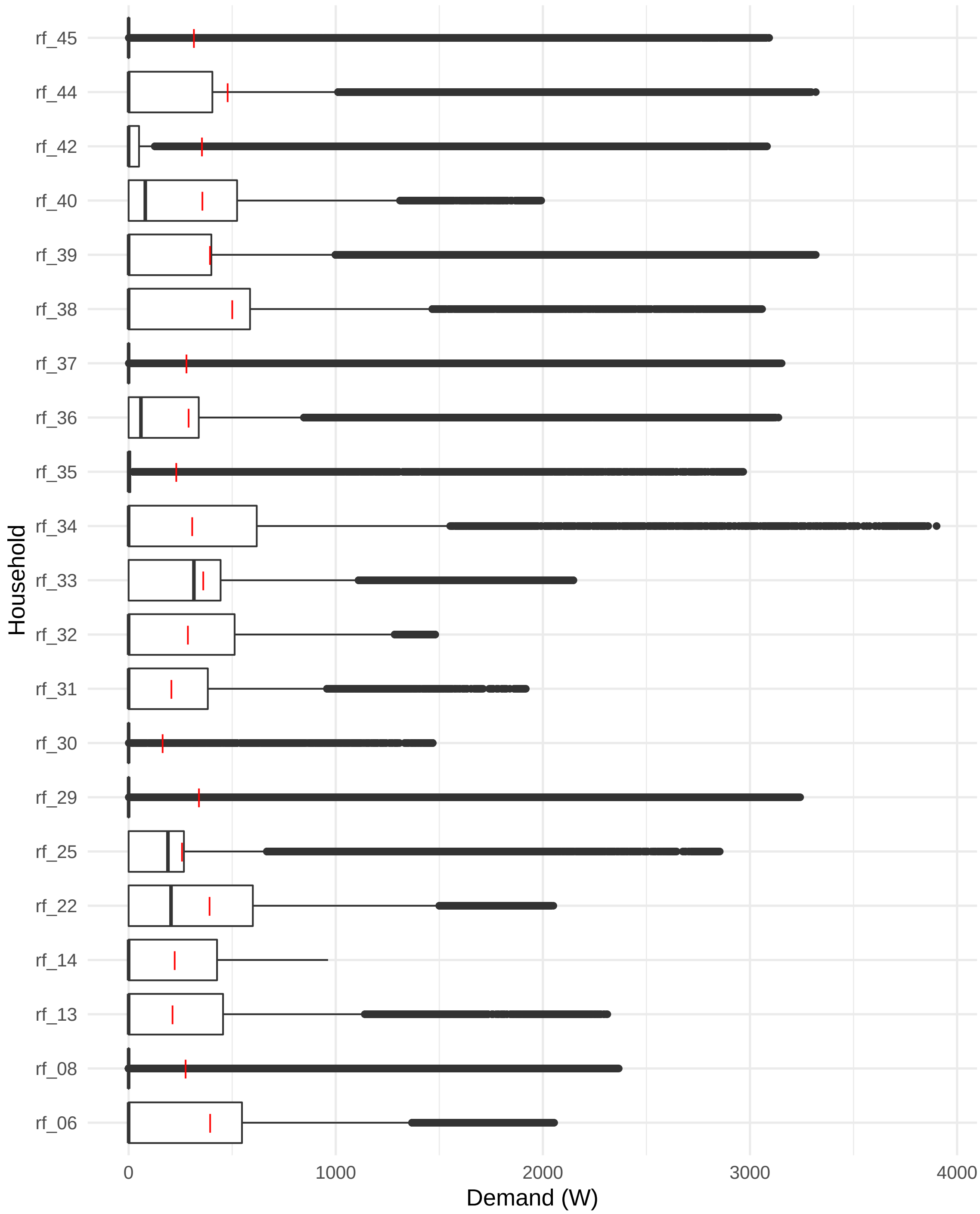

Figure 4.1: Box and whisker plot of hot water electricity demand at half-hourly resolution for sample households

Fig. 4.2 shows the daily hot water electricity demand profile for each household, given by the mean electricity demand over each half hour of the day. It can be seen that these demand profiles vary significantly between households, although many show two distinct peaks, generally during times of day where the grid as a whole experiences peaks. This demonstrates daily patterns of hot water use such as regularly showering in the morning or evening.

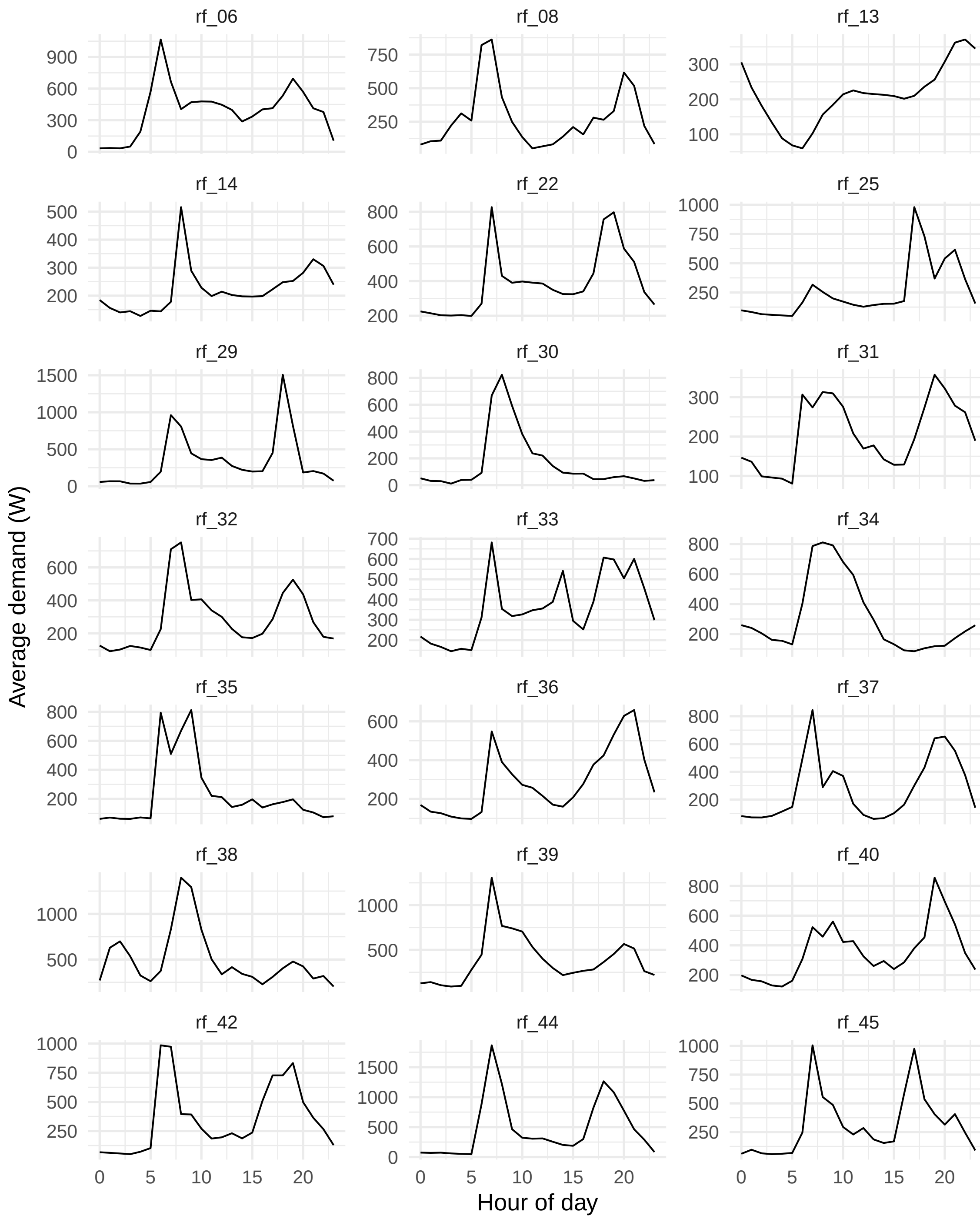

Figure 4.2: Daily profile of hot water electricity demand for sample households at half-hourly resolution

4.2 Autocovariance

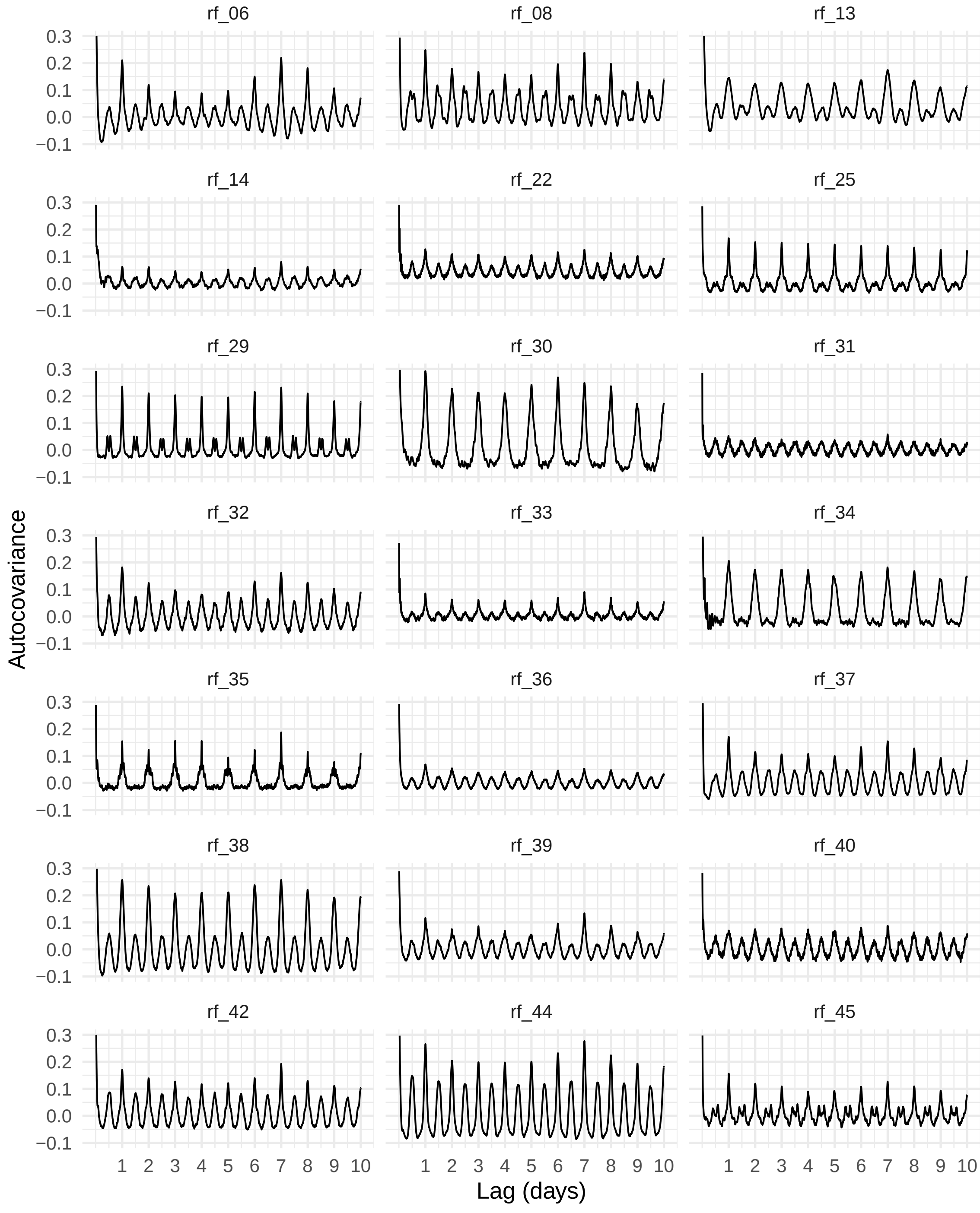

Figure 4.3: Autocovariance of hot water electricity demand for sample households over a 10 day lag

Fig. 4.3 shows the autocovariance (3.7) of our sample households. The peaks at specific lag times show that the households in our sample display strong seasonality, with all households displaying significant peaks in demand occurring on daily intervals, and some households displaying smaller peaks at 12 hour intervals. Some households also display a weekly cycle, which can be seen by the relative increase in the peaks at the seventh day lag.

4.3 Fourier analysis

Using Fourier analysis methods introduced in Section 3.4.3, the three most significant frequencies (corresponding to three individual seasonalities) were determined from the data. Most households had large Fourier frequencies at approximately daily or 12 hourly periods, which align with the autocovariance findings. The three most significant frequencies did not include a weekly period for any of the households. This was contrary to expectations based on existing literature [26] and visual inspection of the autocovariance in Fig. 4.3. For two households, very long frequencies (49 weeks and 132 weeks) were returned as the most significant. Four other households also showed frequencies of greater than one year as the third most significant.

For a list of the three most significant frequencies for sample households as determined by Fourier analysis, refer to Table A.1 in the Appendix.

4.4 Stationarity

A stationary time series does not have expected values or variances that change when applied to different sequential subsets of the data, as described in Section 3.5. As ARMA models (see Section 3.7.7) are more accurate when applied to a stationary time series, hot water electricity demand of each household was tested for stationarity. All of our sample households (except `rf_40`) were determined to be non-stationary. Thus, when constructing ARIMA models (Section 3.7.7), all households except `rf_40` required first-order differencing (see Section 3.7.4), as can be seen by the d values in Table A.3 in the Appendix. This means that for most households, rather than fitting an ARMA model to the original demand data, it was instead fitted to the change in demand between time steps. ARIMAX models (see Sections 3.7.10 and 3.7.11) are slightly less likely to require differencing than the standard ARIMA model, as shown by the d values in Tables A.4 and A.6 in the Appendix. This implies that the errors of the linear regression model are more likely to be stationary than the original data.

4.5 Cross-covariance

Cross-covariance is our method of determining how much hot water electricity demand is correlated with other electricity demand at different time lags (see Section 3.6). If reasonable cross-correlation is found between hot water electricity demand and lagging values of other appliance electricity demand, this relation may be used to predict future values of hot water electricity demand based on current values of other electricity demand.

Fig. 4.4 shows that hot water electricity use in our data set is positively correlated with other electricity use over small time lags. Most households have their maximum cross-covariances occurring at lags of less than half an hour, however correlations at lags of at least half an hour are necessary for half-hour ahead predictions. Some households appear to have their maximum correlation at half hour or one hour lags, and all others show positive correlation for lags up to one hour.

To further explore the optimal time lags for using other electricity demand as a regressor (see Sections 3.7.3, 3.7.10, 3.7.11 and 3.7.12), values of the time lag greater than zero at which cross-correlation is highest were obtained. These were then plotted as a histogram in Fig. 4.5. This figure indicates that for ten households, hot water electricity has a maximum correlation with non hot water electricity within lags of half an hour, and an additional five household have their maximum value between half an hour and an hour. Six households have maximum cross correlation values at lags of greater than one hour.

From these results, we may infer that incorporating the previous two values of other electricity demand into a forecasting model should be sufficient to capture this effect for most households. Models that utilise these results are presented in Sections 5.3, 5.6, 5.8, and 5.9.

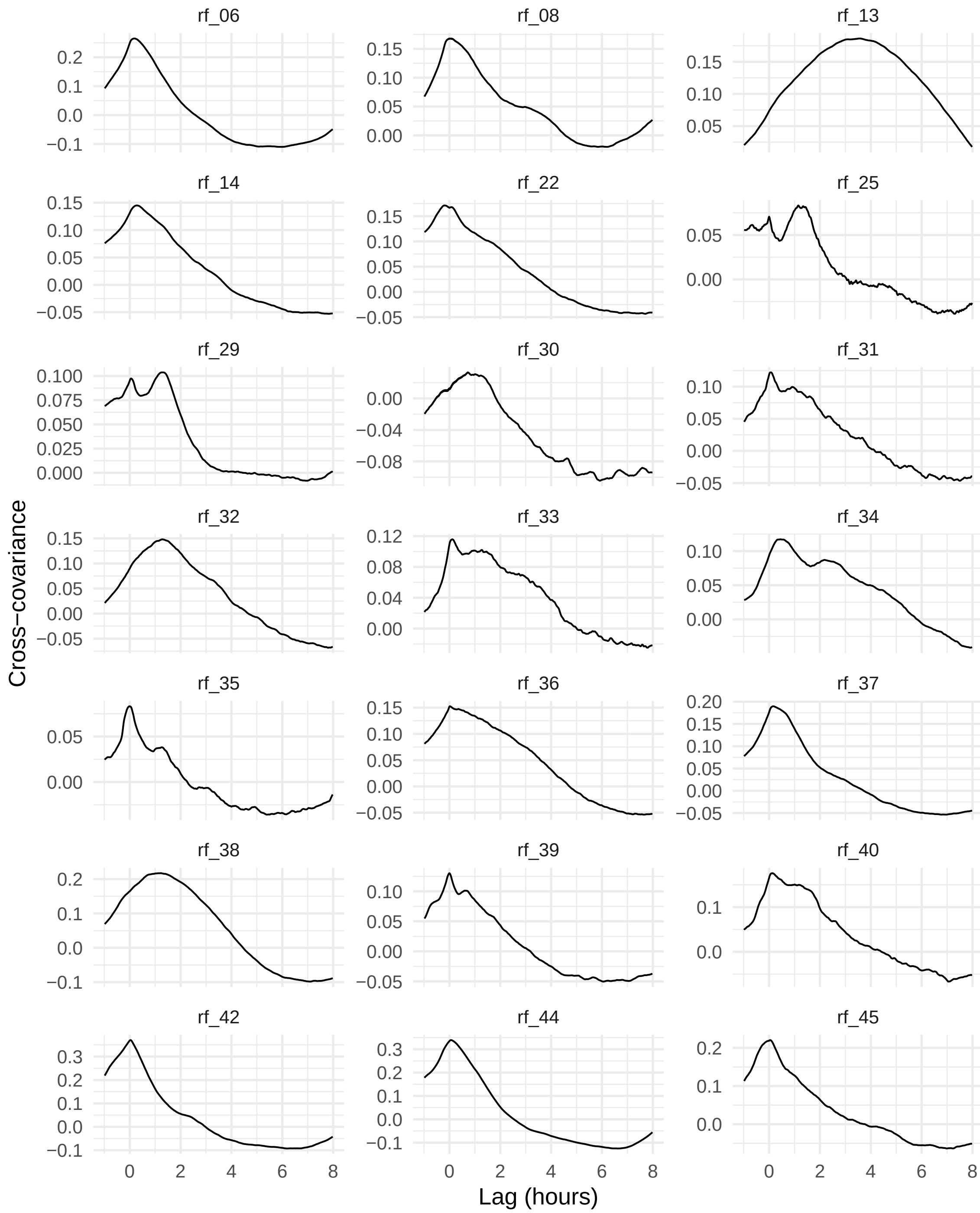

Figure 4.4: Cross correlogram between hot water and other appliances' electricity demand for sample households

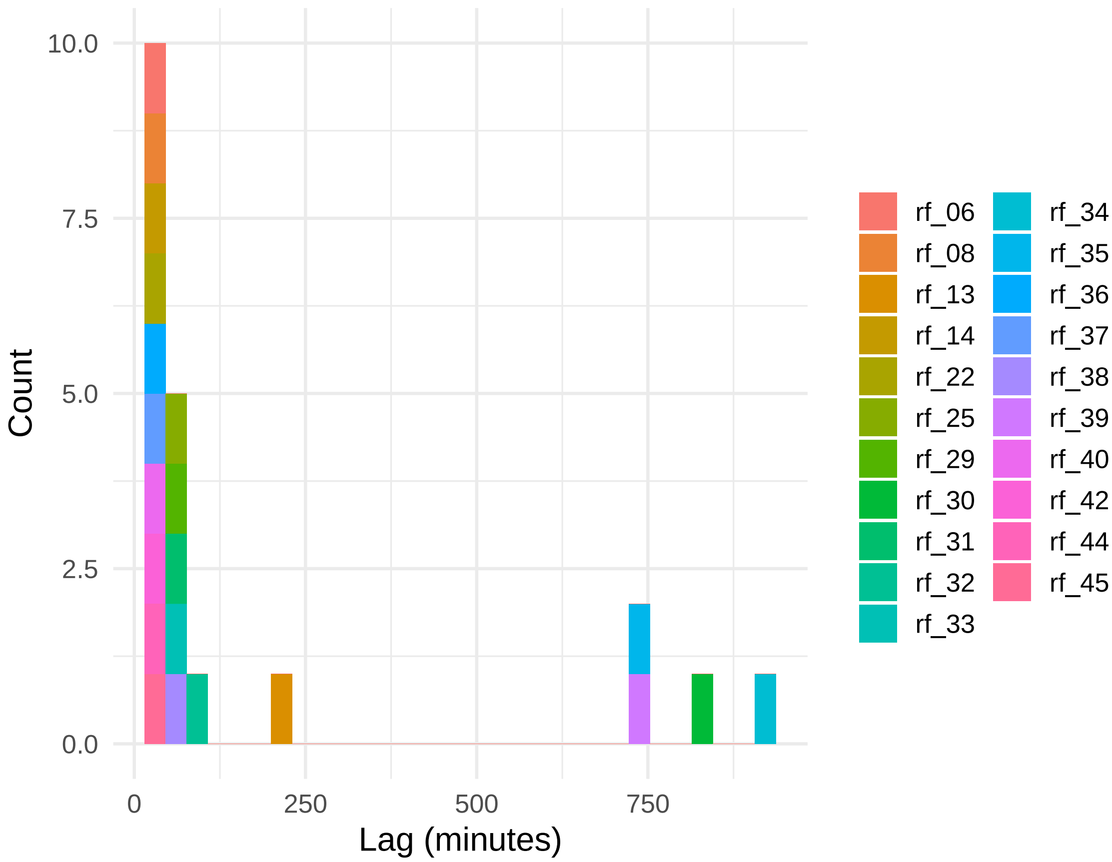

Figure 4.5: Lag at maximum correlation for sample households (bin width 30 minutes)

4.6 Linear Regression

As an exploratory technique, simple linear regressions were plotted between hot water demand and the demand of other appliances with a half hour time lag (an example is provided in Fig. 4.6).

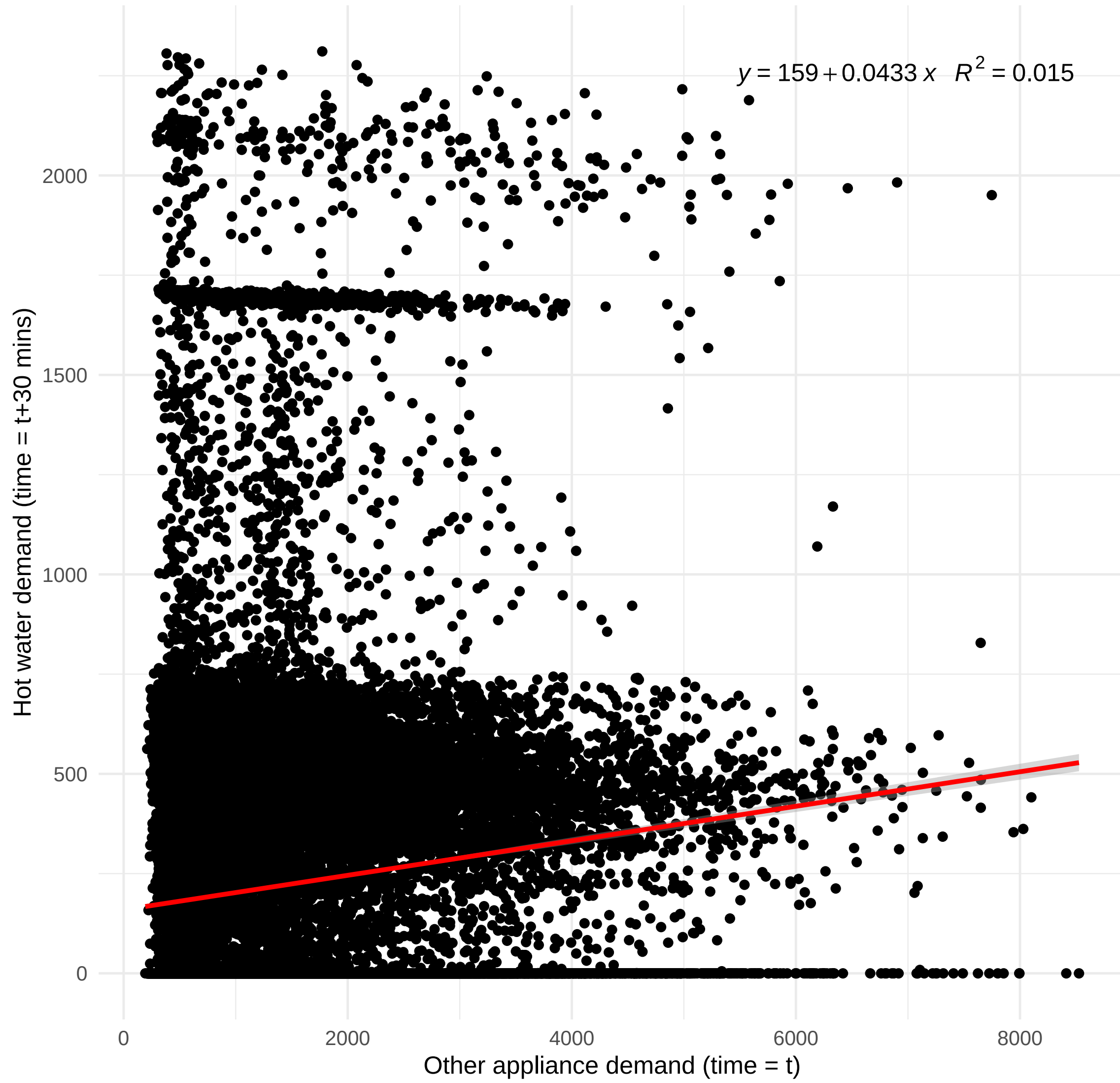

Figure 4.6: Linear regression between hot water electriciy demand and electricity demand of other appliances with a half hour time lag for household rf 13)

Small positive correlations were found for all households.

4.7 Implications for model building

This chapter presented results from some key methods used to understand patterns within the time series data. Sections 4.2 and 4.3 showed that hot water electricity demand displays daily seasonality. This suggests that increased accuracy would be attained by forecasting models which can take seasonality into account. Section 4.5 showed that hot water electricity demand correlates with previous values of other appliances' electricity demand. The maximum correlation between hot water electricity demand and lagged other electricity demand occurs within the first hour for most households. Thus, when incorporating other appliance demand into forecasting models, we use the previous two time steps to best capture this correlation. The following chapter displays the results obtained from forecasting models that incorporate these components of the data.

Chapter 5

Results

Existing research in Chapter 2 provided a guide to which data analysis techniques would be effective for achieving the aims of this thesis. Chapter 3 described these techniques in more detail. Chapter 4 presented results from preliminary data analyses, and noted the implications of these results on forecasting models. Using these preliminary results, the forecasting models described in Section 3.7 were fitted to the set of training data (see Section 3.8) for each household. The ability of these models to predict hot water half hour in advance was then tested against the set of validating data. This chapter presents the results of these predictions, with the performance of each model evaluated against the metrics outlined in Section 3.10.

5.1 Naive model

As discussed in Section 3.7.1, we use a simple random walk as a naive model by which to compare the performance of other models. When used to predict the next time step, the random walk given in (3.13) becomes $\hat{x}_{t+1} = x_t$. In Fig. 5.1, we show an example plot of the model's prediction of demand compared to actual demand. This method is reasonably effective for some houses, in particular those that have long periods where the element power has roughly the same output. In contrast, households that exhibit frequent oscillation of element power are very poorly modelled by this method, with the predicted values consistently 'missing' the actual values.

The naive model has RMSEs ranging between 312W and 886W, with the average over all households being 602W. During peak periods, RMSEs ranged between 343W and 1192W, with the average over all households being 760W. Examination of the daily profile of residuals (see Fig. 5.2) shows that this model performs quite poorly during peak periods, with RMSEs during peak periods being 26.4% greater than average RMSEs. The autocovariance of residuals in Fig. 5.3 show negative initial correlation that decays reasonably quickly, with many households having periodic spikes. This indicates that the naive model does not effectively capture seasonal properties of the data.

The model precisely mimics the actual data with a half-hour lag, and therefore scores highly for physical fidelity. Interpretability is high as this model is represented by a very simple equation, although no illumination as to underlying behavioural properties are actually provided by this model. The computational times to fit models to each household were negligible for this model.

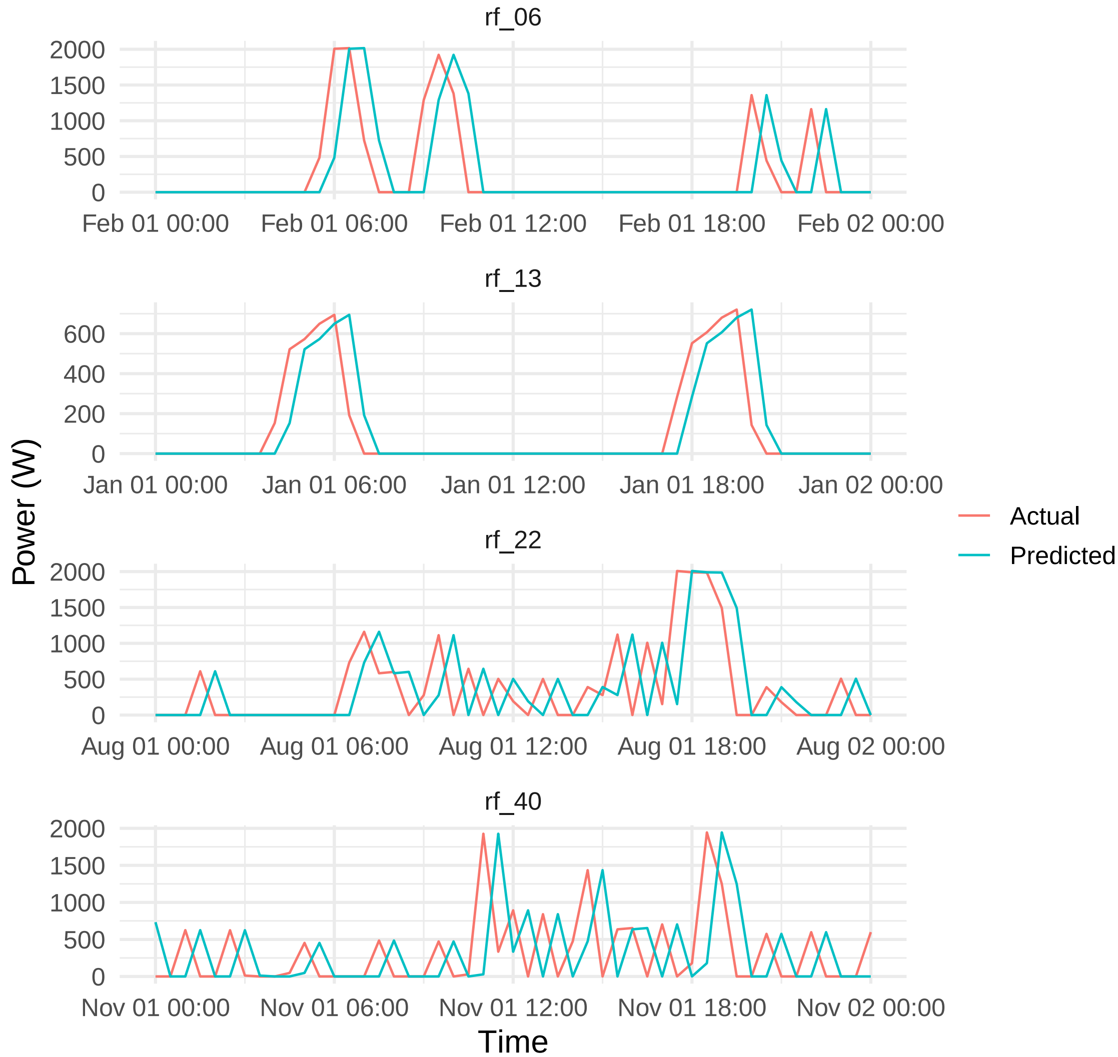

Figure 5.1: Performance of the naive model for four households over four separate days

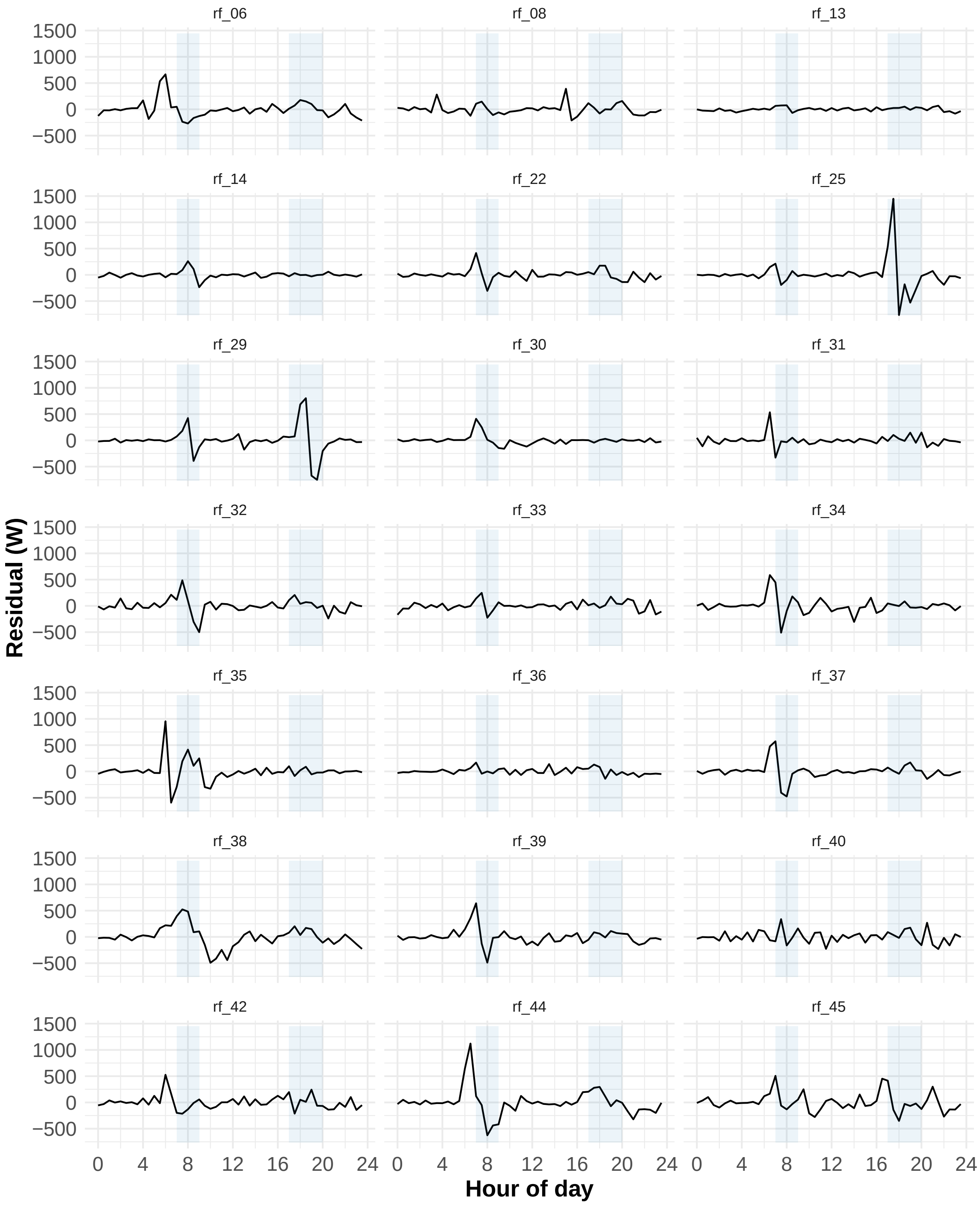

Figure 5.2: Daily profile of residuals for the naive model (peak periods shaded)

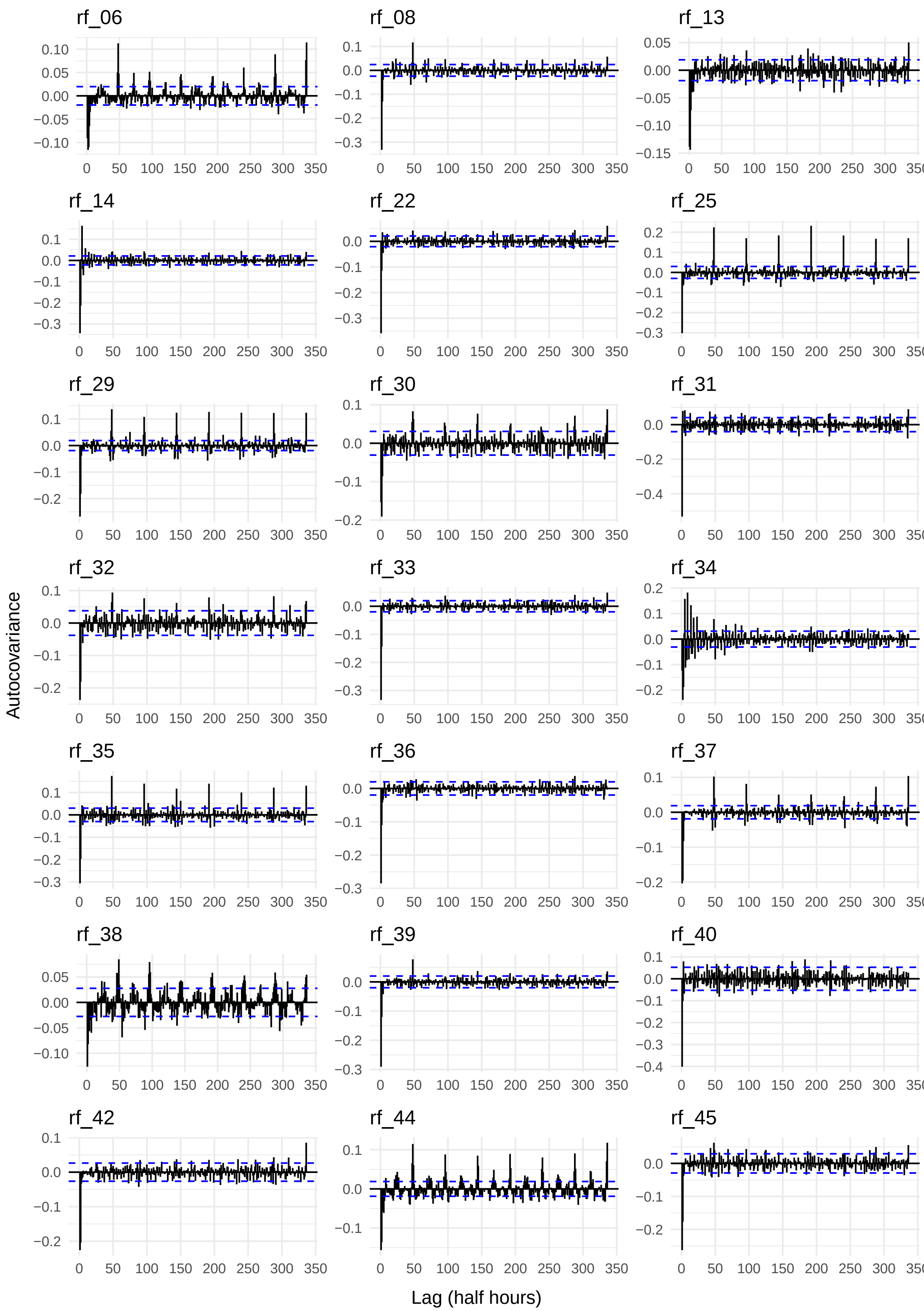

Figure 5.3: Autocovariance of residuals for the naive model

5.2 Seasonal naive

Similarly to our naive model, the seasonal naive model (see Section 3.7.2) predicts one (half hour) time step onto the future utilising only one prior observation. Where the seasonal naive differs is that it assumes the most likely value for hot water demand will be the value at that same time one period prior. Based on the results in Section 4.2 that demonstrate daily periodicity, we chose the period length to be one day, thus (3.14) becomes $\hat{x}_t = x_{t-48}$. The performance of this model is demonstrated in Fig. 5.4.

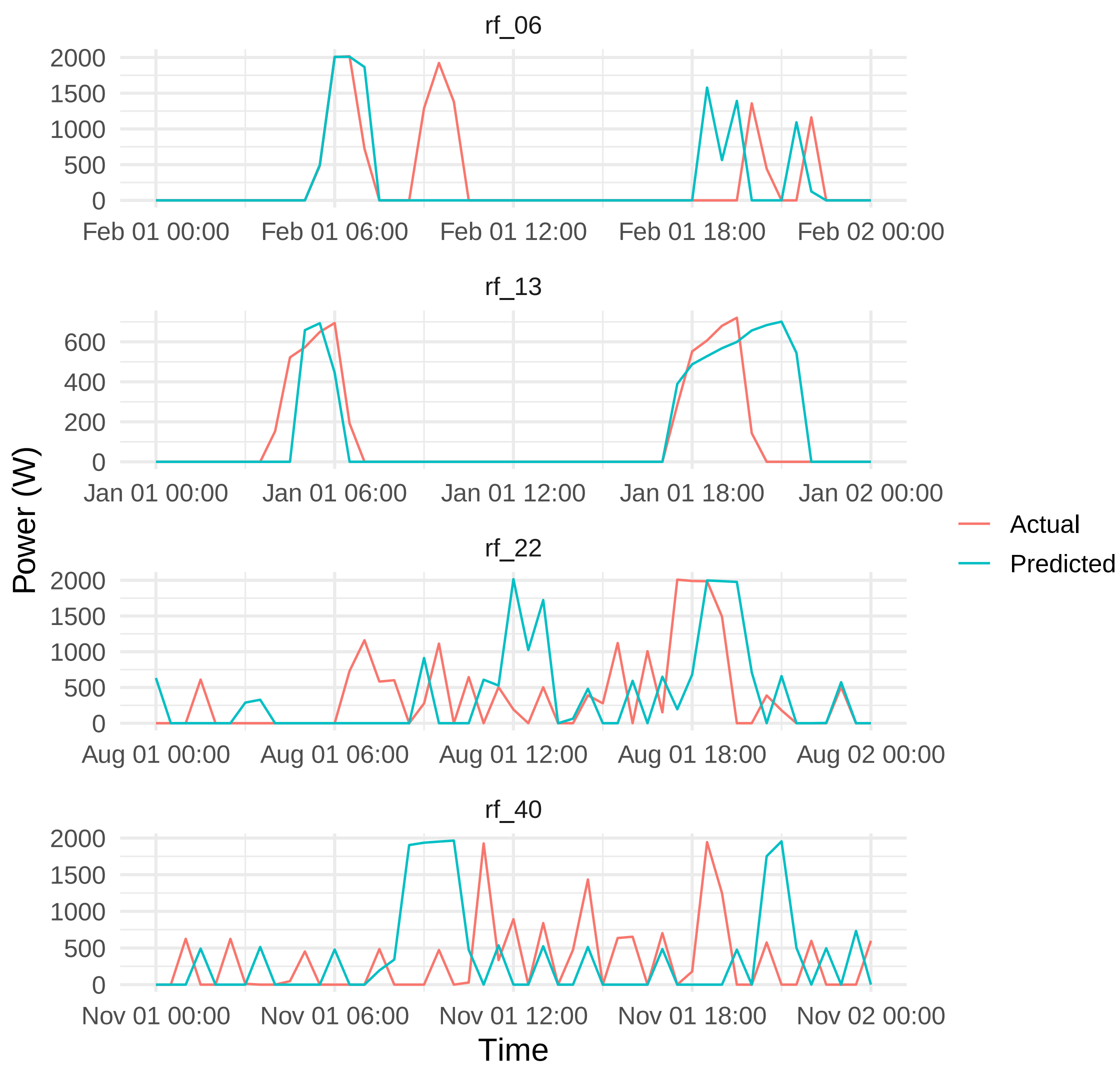

Figure 5.4: Performance of the seasonal naive model for four households over four separate days

In general this method is quite ineffective, although in some instances it makes very accurate predictions of the initial instance of the element turning on. Contrary to the results in [26], the seasonal naive model was less accurate than the naive model. The reason for this discrepancy may lie in the data itself, with the UK households having more weekly regularity than the NZ households used in this research. Alternatively, this discrepancy may lie in the differences in data resolution or forecast length. The seasonal naive model has RMSEs ranging between 396W and 1221W. The average RMSE over all households is 729W, corresponding to a 21% decrease in accuracy compared to the naive model.

Despite attempting to consider seasonality, this model performs very poorly during peak periods. During peak periods, RMSEs ranged between 447W and 1647W, with the average over all households being 927W. This corresponds to a PEI of 27.1%, slightly higher than that of the naive model. This is indicated in the daily profile of residuals in Fig. 5.5. The autocovariance of residuals in Fig. 5.6 show most households decaying reasonably quickly, with less periodicity than the naive model. This indicates the seasonal naive model is effectively capturing underlying properties of the data. However, a large negative autocorrelation occurs at the selected seasonal period of 48 hours, and there are still occurrences of values greater than the significance level.

The model precisely mimics the actual data with a week long lag, and therefore scores highly for physical fidelity. Interpretability is high as this model is represented by a very simple equation, although the only illumination this model provides into behavioural properties of hot water use are with respect to how strictly a household follows weekly cycles of demand through examination of the residuals. The computational time to fit models to each household was negligible for this model.

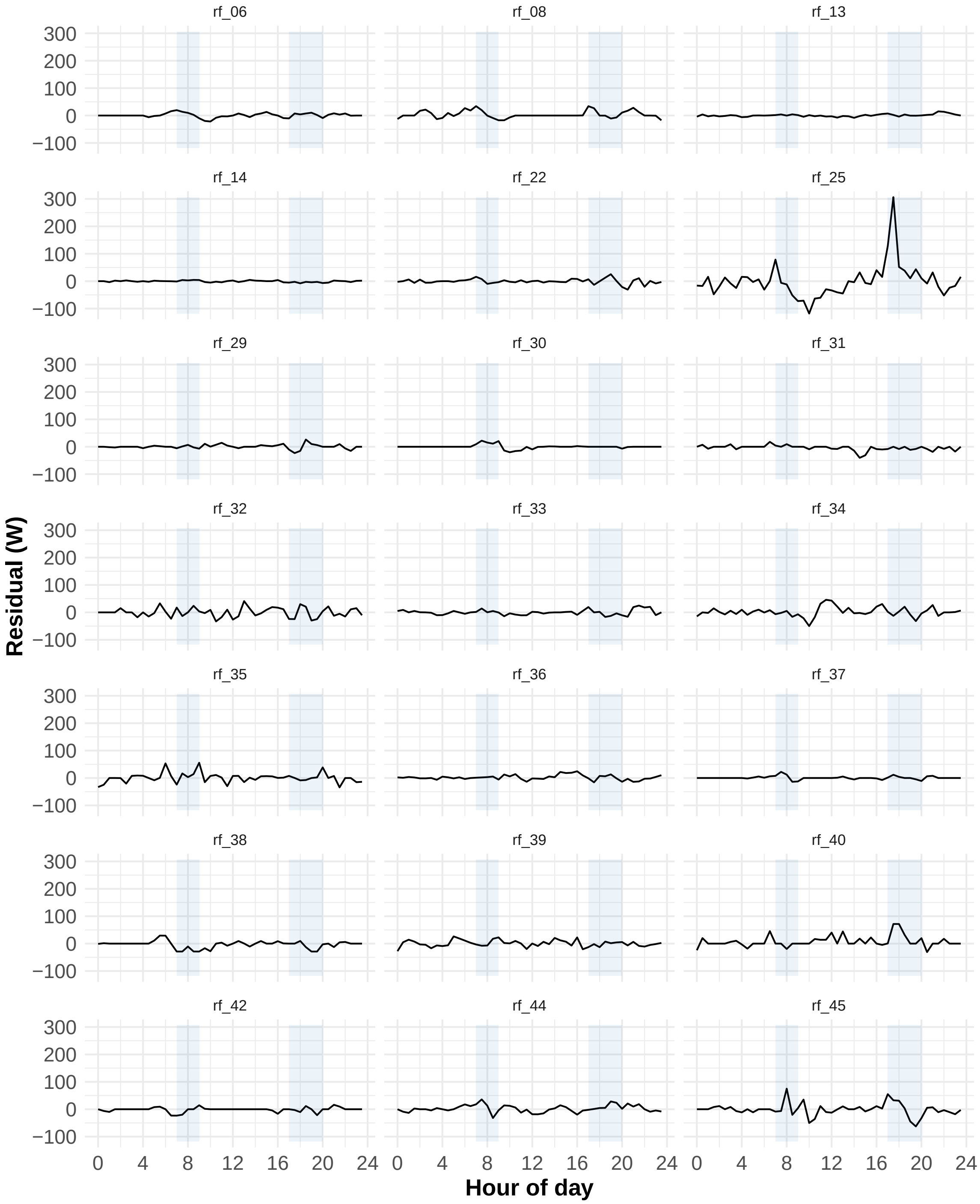

Figure 5.5: Daily profile of residuals for the seasonal naive model (peak periods shaded)

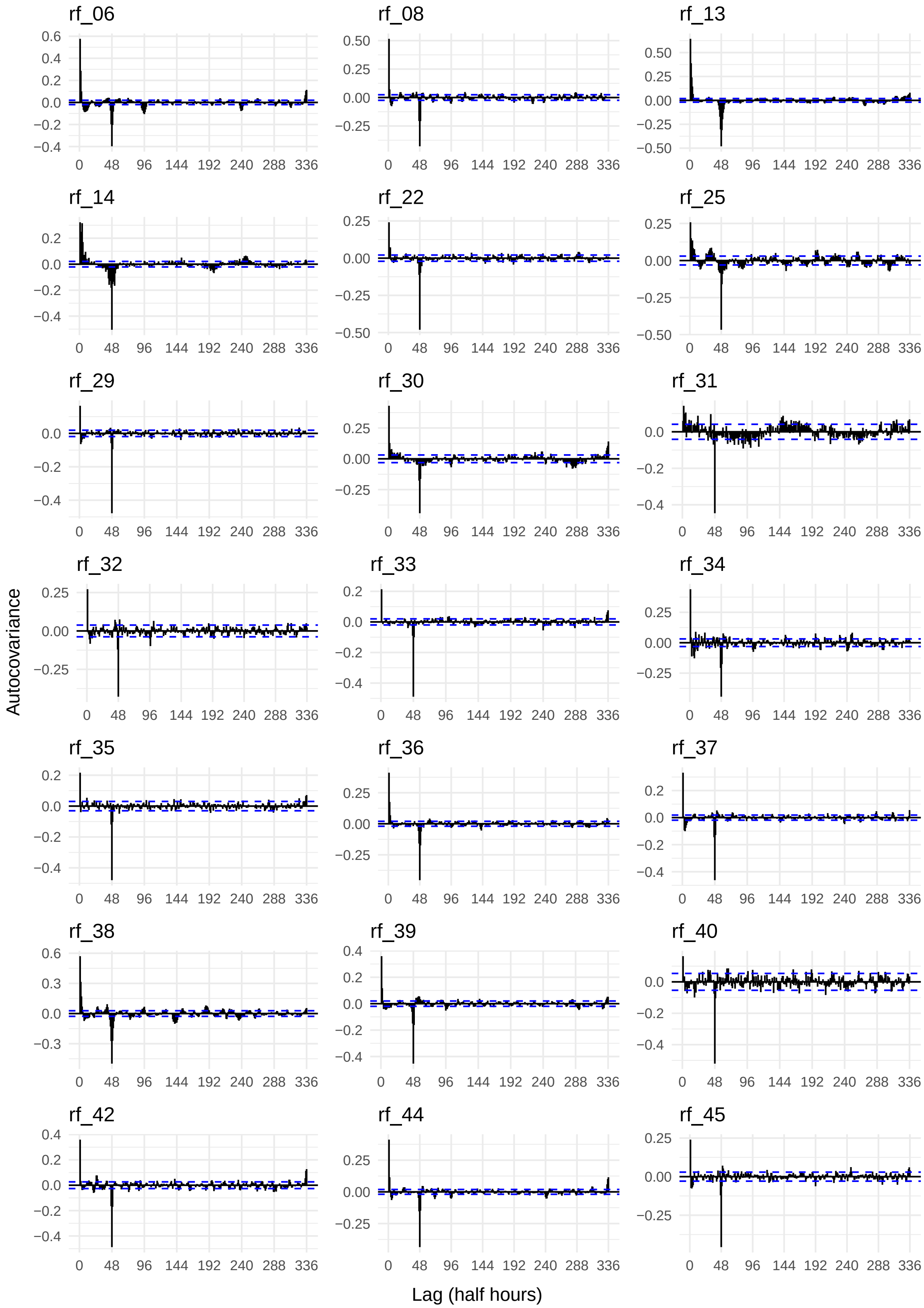

Figure 5.6: Autocovariance of residuals for the seasonal naive model

5.3 Simple linear regression

This model uses (3.15), along with the results regarding optimal time lags for cross-covariance obtained in Section 4.5. From these, a simple linear model is constructed which forecasts hot water electricity use based on the previous two half-hour values of other appliance electricity use, i.e.

$$\hat{x}_t = \gamma_0 + \gamma_1 y_{t-1} + \gamma_2 y_{t-2}. \tag{5.1}$$

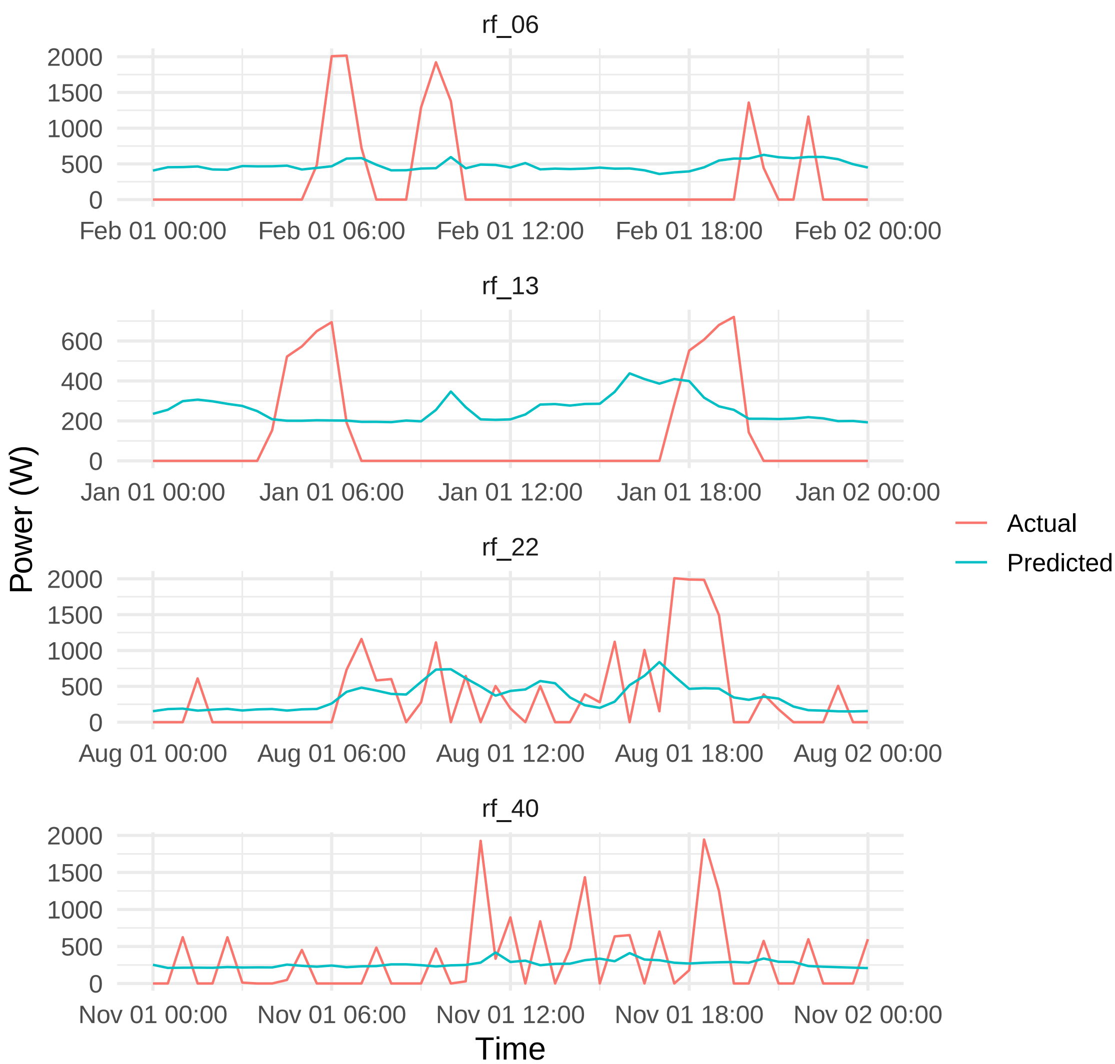

Figure 5.7: Performance of the simple linear regression model for four households over four separate days

Fig. 5.7 shows the performance of this model for four separate households. The simple linear regression model has RMSE ranging between 307W and 1000W, with the average over all households being 578W. This corresponds to a 4% increase in accuracy compared to the naive model. During peak periods this model performed particularly poorly, as can be seen in Fig. 5.8. Peak-time RMSEs ranged between 333W and 1289W, with the average over all households being 761W. This corresponds to a PEI of 31.6%. The autocovariance of residuals in Fig. 5.9 show very strong periodicity, indicating that this model is not effectively capturing seasonality of data.

The model does not accurately capture zero values present in the actual data, and is also significantly smoother than the actual data, however no negative values are present. It scores low for physical fidelity. Interpretability is high, as the output can be understood easily by (5.1). The computational time to fit models is negligible.

While this model would not be a very effective means of predicting hot water electricity demand on its own, it does outperform the naive model in accuracy. This suggests that linear regression of other electricity demand may be effective as an input into more complex models that take other aspects of the data into consideration, as we see in the results for the ARIMAX and STL + ARIMAX models in Sections 5.6 and 5.8 respectively.

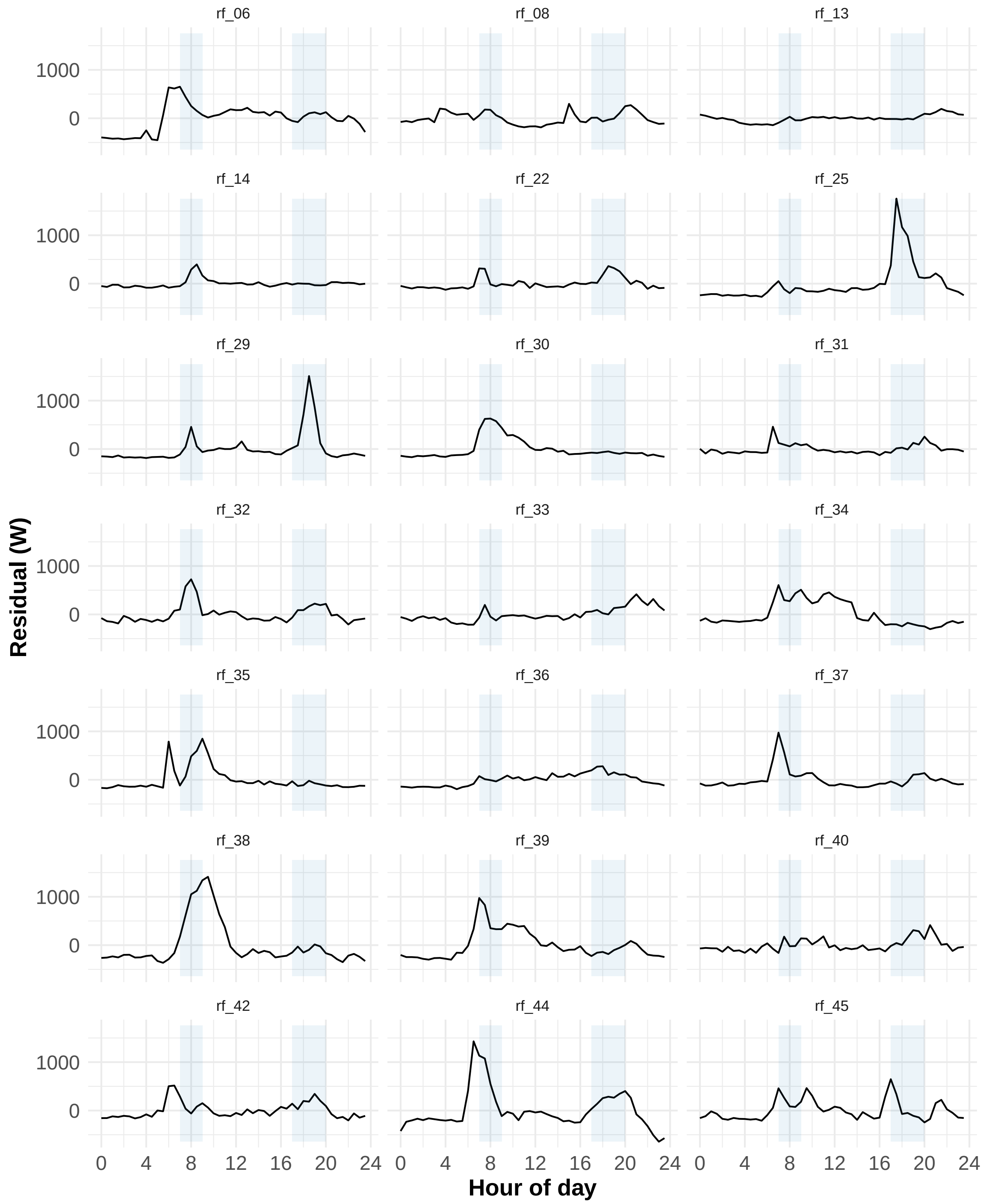

Figure 5.8: Daily profile of residuals for the simple linear regression model (peak periods shaded)

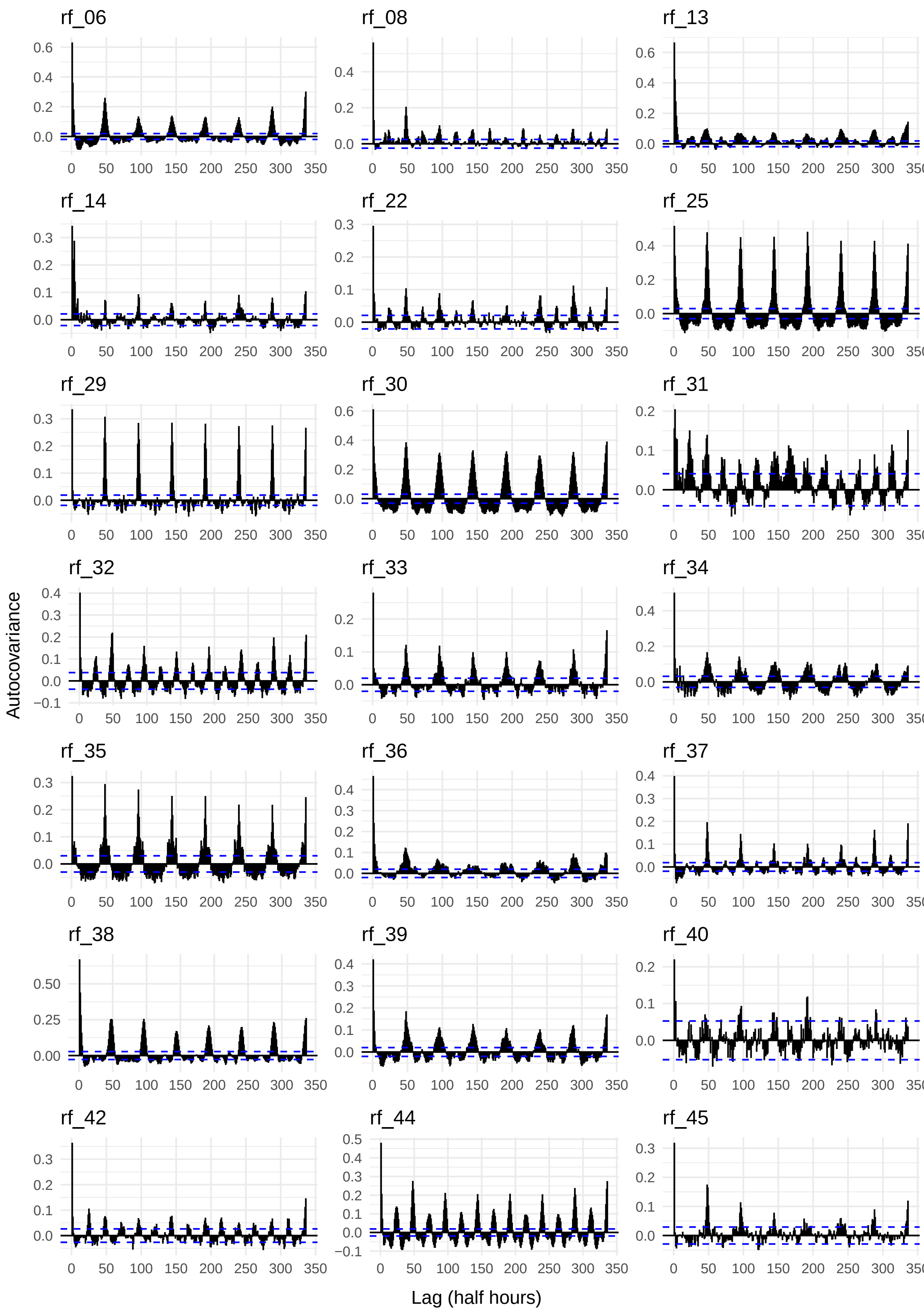

Figure 5.9: Autocovariance of residuals for the simple linear regression model

5.4 ARIMA

In accordance with the procedure outlined in Section 3.7.7, ARIMA models were fitted to data from each household. As each household has different usage patterns and characteristics, optimal values for parameters p and q differ for each household [refer to (3.22) or (3.23)]. Parameters are selected in order to minimise the AIC [see (3.24)], and are provided in Table A.3 in the Appendix. As mentioned in Section 4.4, all households except `rf_40` required first-order differencing ($d = 1$) to become stationary.

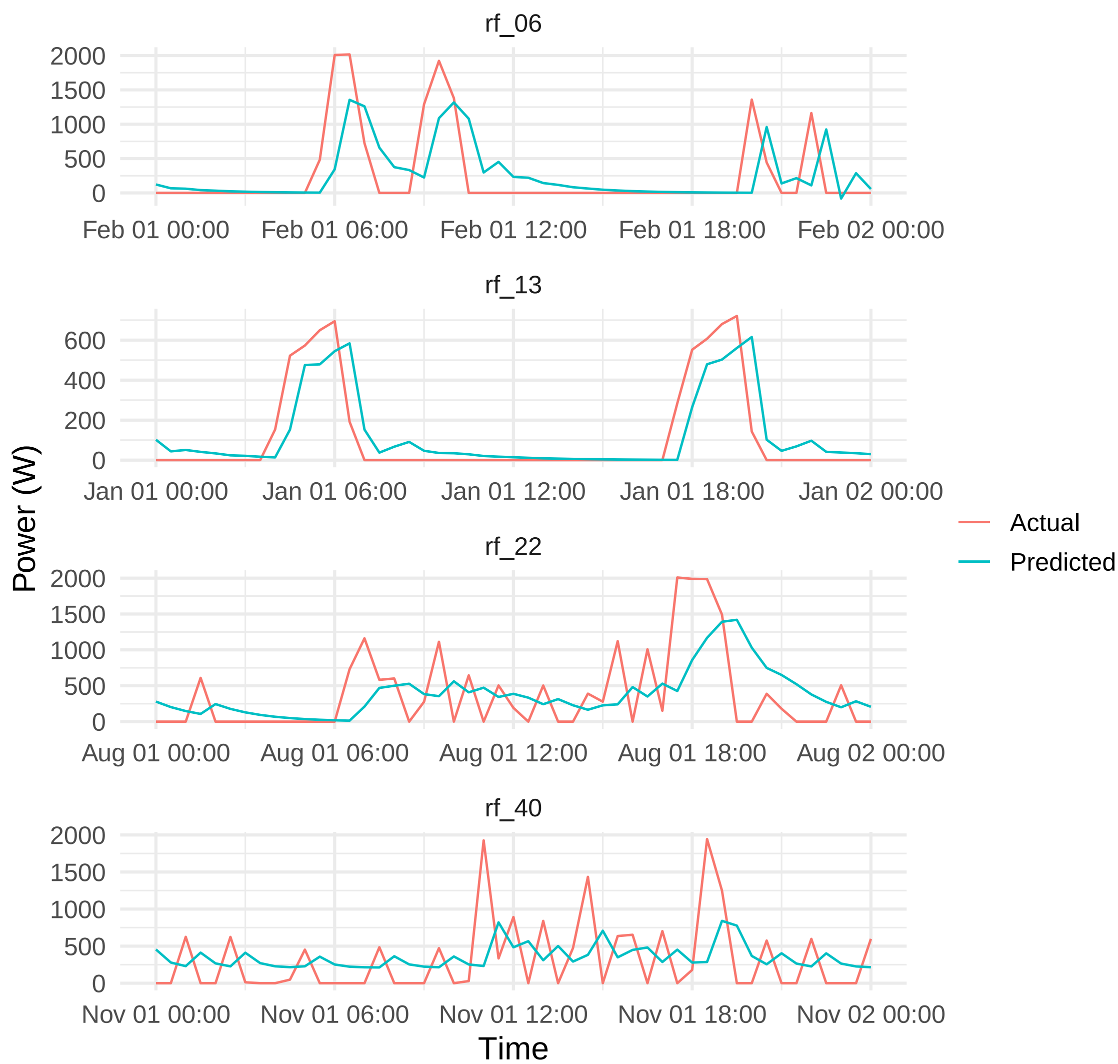

Figure 5.10: Performance of the ARIMA model for four households over four separate days

The ARIMA model has RMSEs ranging between 294W and 864W, with the average over all households being 553W. This corresponds to a 8% increase in accuracy compared to the naive model. During peak periods, RMSEs ranged between 327W and 1191W, with the average over all households being 711W. This corresponds to a peak period RMSE increase of 28.5%. This underperformance during peak periods is reiterated in the daily profile of residuals in Fig. 5.11. The autocovariance of residuals in Fig. 5.12 show many households with strong residual periodicity, indicating that this model is not effectively capturing seasonality of data. However, the low and rapidly decaying initial values indicate that autoregressive properties of the data are sufficiently captured.

When observing the general shape of the model prediction in Fig. 5.10, we see that the model returns values that are somewhat smoother than the actual data. However, this effect is far less pronounced than that of the linear regression model (see Fig. 5.7). Occasionally, the model predicts negative values or values greater than the element is capable of outputting. While the shape of the model is smoother than the data, for many households it is reasonably similar. This model therefore scores low to moderate for physical fidelity. Interpretability is moderate, as, given familiarity with the model, the output can be understood by the value of the p, d and q parameters. For a first order differenced ARIMA model ($d = 1$), the autoregressive component p uses a linear regression from the most recent p changes in demand to predict the following change in demand [see (3.17)]. Similarly, the moving average component of a first order differenced ARIMA model uses a linear regression from the previous q residuals in the model to predict the following change in demand [see (3.21)].

The computational time to fit models to each household varied between 0.17s and 4.47s. The average fitting time over all households is 1.27s. While being non-negligible, these fitting times would be tolerable even when fitting models to hundreds of thousands of households.

Figure 5.11: Daily profile of residuals for the ARIMA model (peak periods shaded)

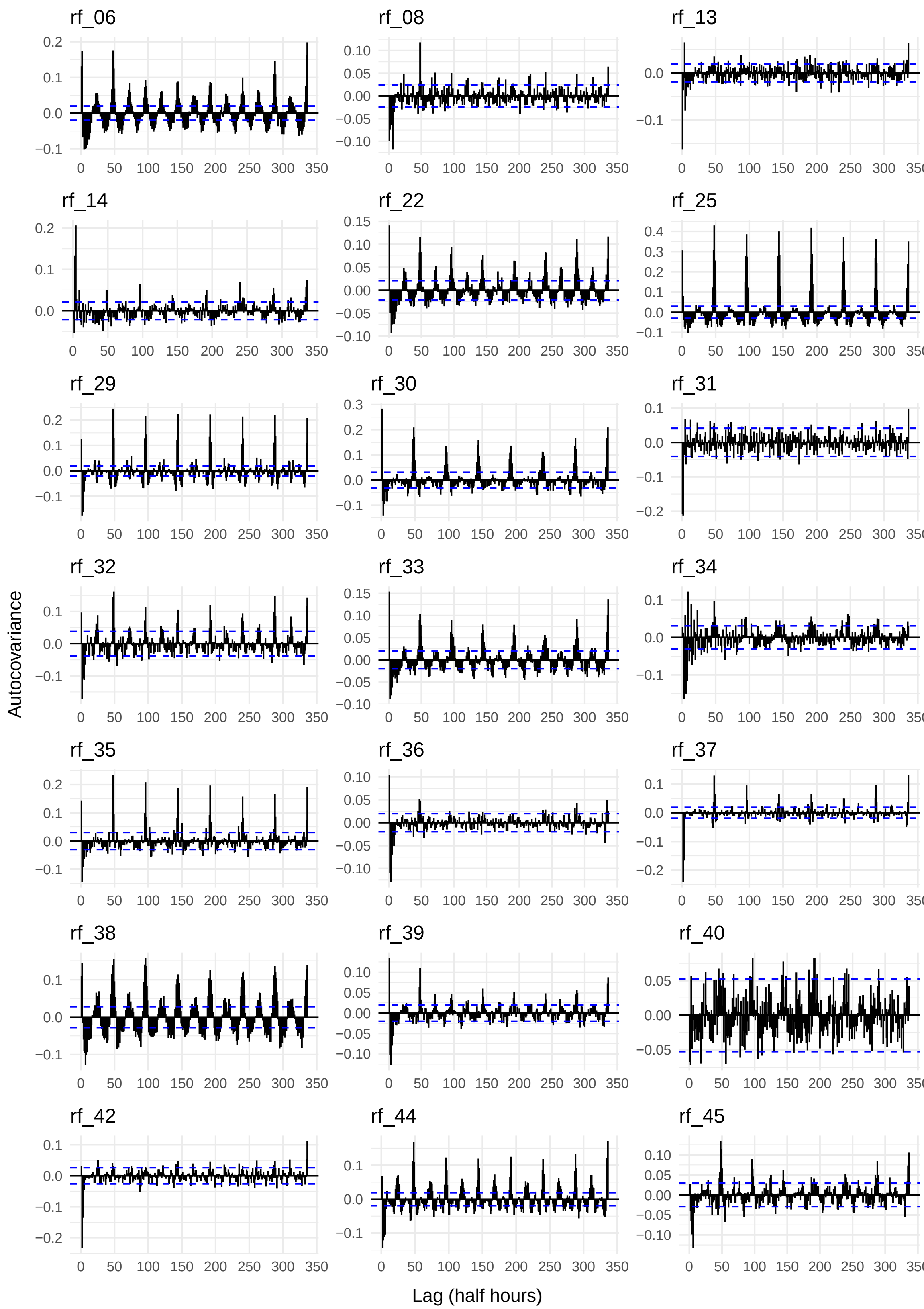

Figure 5.12: Autocovariance of residuals for the ARIMA model

5.5 Seasonal ARIMA

The periodic nature of hot water electricity demand described in Section 4.2 suggests that the seasonal ARIMA model would provide more accurate predictions than the standard ARIMA model for this data. In addition, as this model falls under the 'conventional' forecasting category, it would be reasonable to assume it was computationally efficient. However, preliminary modelling found that the seasonal ARIMA model took an extremely long time to fit, with households taking upwards of 8 hours each to have this model fitted to their data.

The seasonal ARIMA model was originally designed to remove annual seasonality, often from financial or economic data at monthly or 3-monthly resolution. Perhaps for this reason, the fitting algorithm is not optimised to handle data at half-hour resolution. Significant effort was made to improve computational speed of this model, however no progress was made. The seasonal ARIMA model was therefore not pursued any further. Seasonality of data was instead taken into consideration within an ARIMA model through the STL method, the results of which are presented in Section 5.7.

5.6 ARIMAX

As described in Section 3.7.10, the ARIMAX model may be considered as a linear regression model with an ARIMA model fitted to its errors. In accordance with the results in Section 4.5, the previous two values of other electricity demand were included as the regressors. Thus (3.26) becomes

$$\hat{x}_t = \gamma_1 y_{t-1} + \gamma_2 y_{t-2} + \alpha_1 x_{t-1} + \alpha_2 x_{t-2} + ... + \alpha_p x_{t-p} + \beta_1 w_{t-1} + \beta_2 w_{t-2} + ... + \beta_q w_{t-q}. \quad (5.2)$$

Optimal values for parameters p, d, and q for the ARIMA modelled errors (see Section 3.7.7) differ for each household, and are provided in the Appendix (Table A.4).

The ARIMAX model has RMSEs ranging between 293W and 850W, with the average over all households being 542W. This corresponds to a 10% increase in accuracy compared to the naive model. During peak periods, RMSEs ranged between 330W and 1148W, with the average over all households being 694W. This corresponds to a peak-time increase in RMSE of 28.2%, indicating that the addition of the external regressors assists in peak-time accuracy when compared to the ARIMA model. This may be seen by comparing daily profiles of residuals in Fig. 5.14 by those of the ARIMA model in Fig. 5.11. It can also be seen by comparing the autocovariance of residuals provided in Fig. 5.15 by the autocovariance of ARIMA residuals in Fig. 5.12. While the addition of the external regressor provides some reduction in periodic behaviour of the autocorrelation function of residuals, Fig. 5.15 shows that seasonality of the data is still not being adequately captured in this model.

The ARIMAX model shows similar behaviour to the ARIMA model with respect to the predictions being smoother than the actual data, as may be seen in Fig. 5.13. Occasional negative values or values above that capable by the element occur. It scores low to moderate for physical fidelity. Interpretability is moderate, as the output can be understood by the values of the p, d and q parameters, along with the values given to the regressors. The computational time to fit models to each household varied between 0.71s and 13.97s, with the average over all households taking 4.08s.

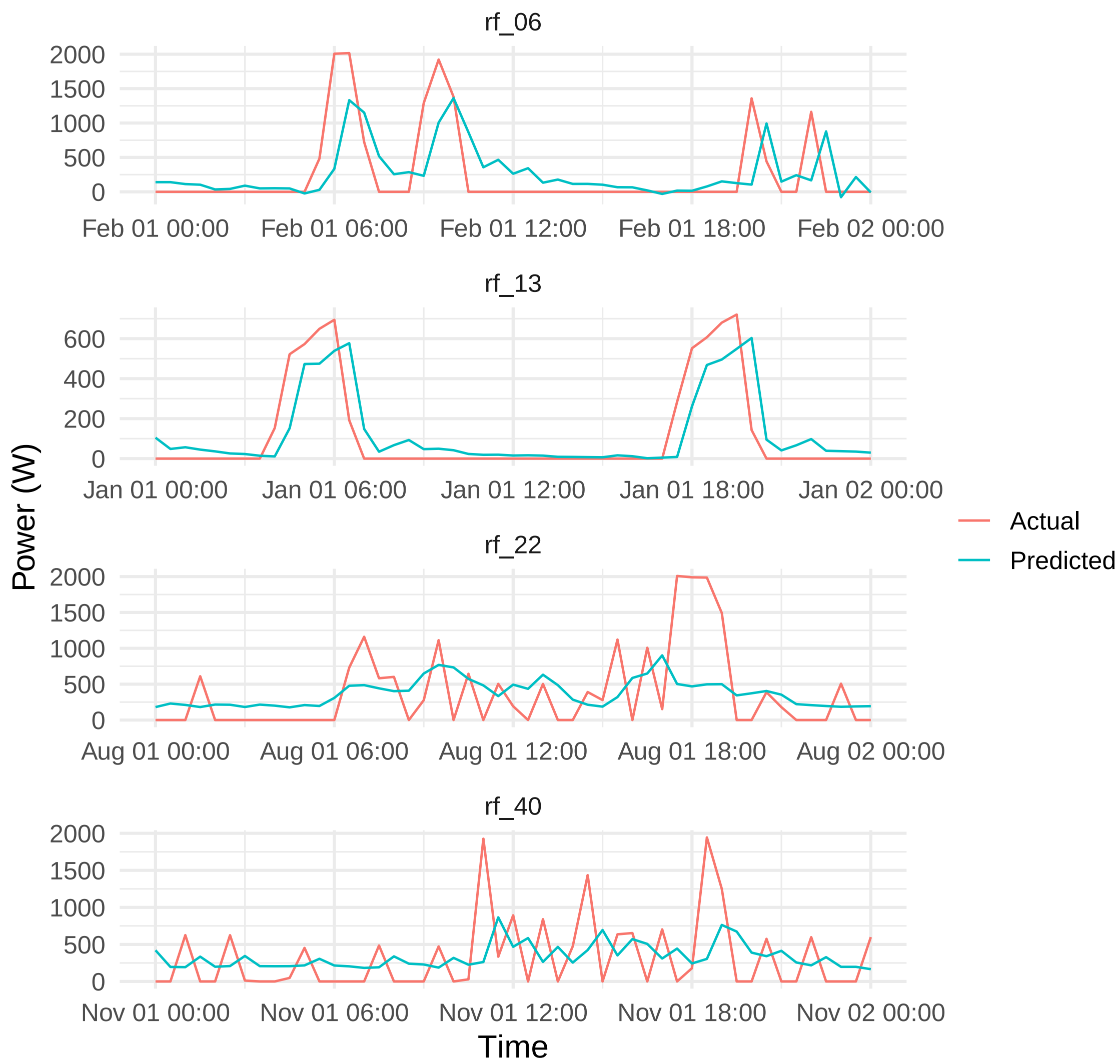

Figure 5.13: Performance of the ARIMAX model for four households over four separate days

Figure 5.14: Daily profile of residuals for the ARIMAX model (peak periods shaded)

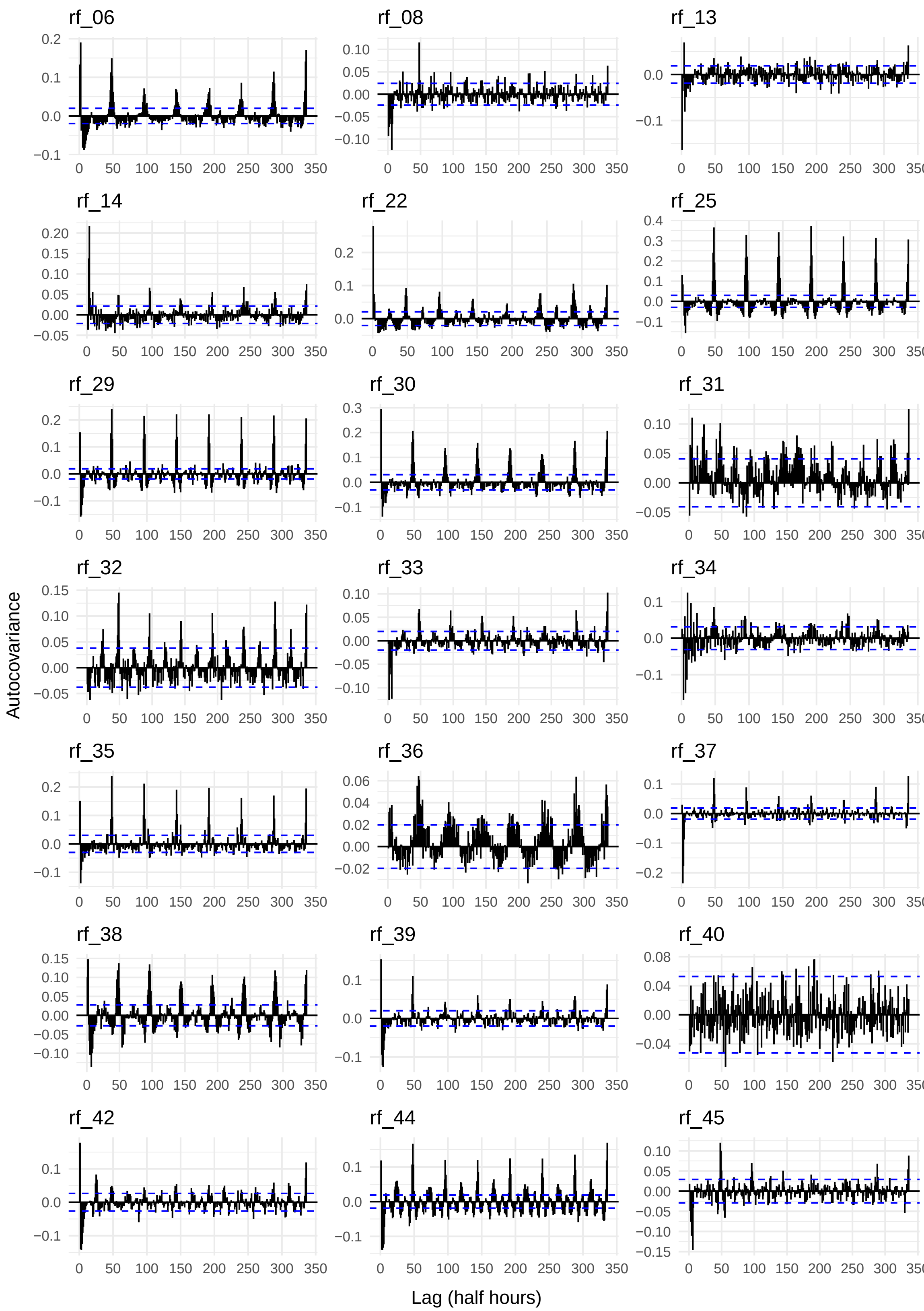

Figure 5.15: Autocovariance of residuals for the ARIMAX model

5.7 ARIMA with STL decomposition

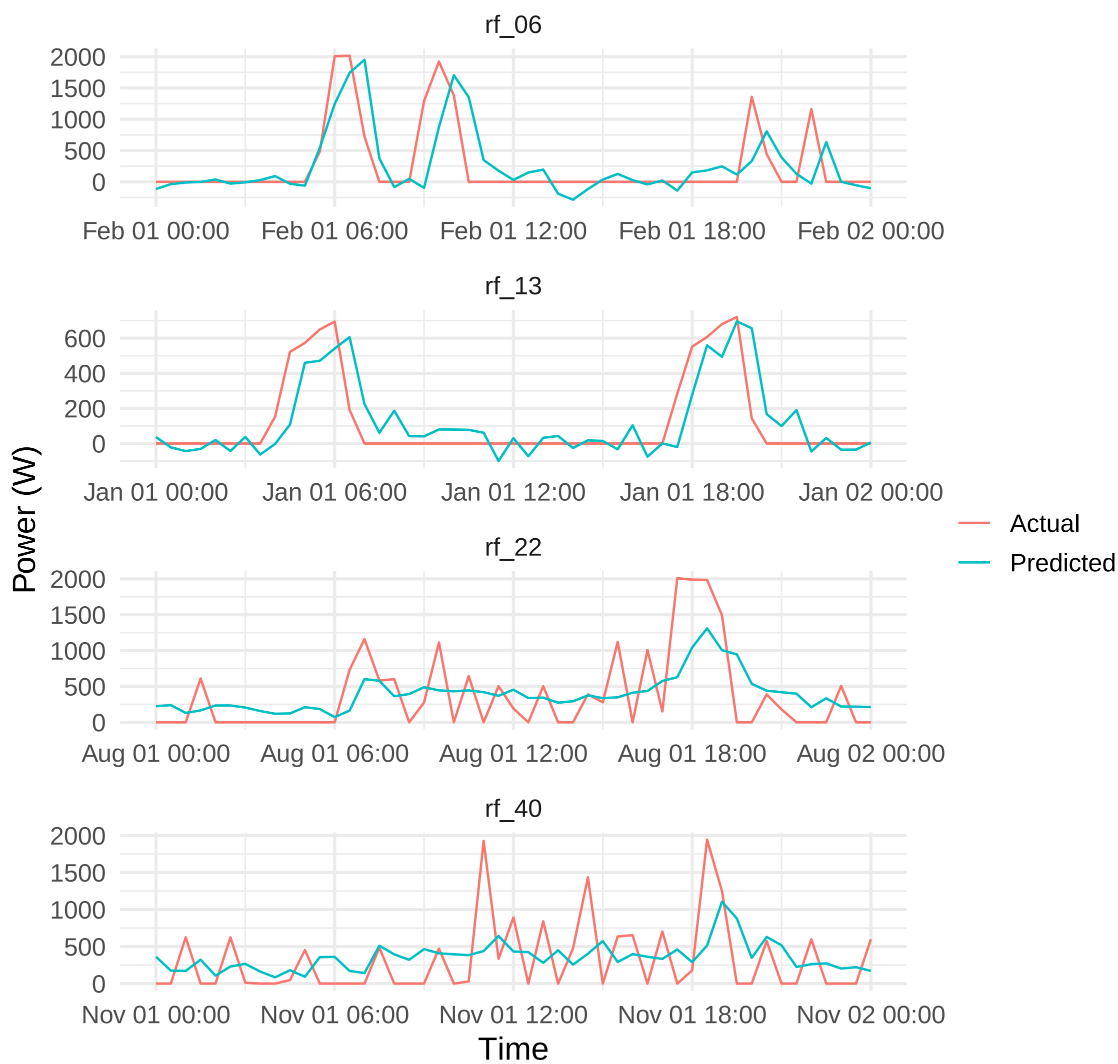

Figure 5.16: Performance of the STL + ARIMA model for four households over four separate days

In accordance with the methods in Section 3.7.8 and the results from Chapter 4, ARIMA models were fitted to household data that had been decomposed using the STL method. Parameters for the ARIMA component of the model are provided in Table A.5 in the Appendix. Performance of these models for four households are shown in Fig. 5.16.

Our STL + ARIMA models had RMSEs ranging between 278W and 784W, with the average over all households being 514W. This corresponds to a 14% increase in accuracy compared to the naive model. During peak periods, RMSEs ranged between

292W and 1106W, with the average over all households being 644W. While still losing accuracy during peak periods, with a PEI of 25.1%, this model had the lowest PEI of all those considered in this thesis. This is further demonstrated by the daily profile of residuals provided in Fig. 5.17. The autocovariance of residuals is provided in Fig. 5.18. These show reasonably rapid decay with limited periodic spikes beyond the significance level, indicating that this model is effectively capturing underlying properties if the data.

Note that this model was the most accurate of a selection of time series methods used to forecast volumetric residential hot water demand in [26]. Our results come to the same conclusion, with further accuracy improvements only achieved in this research by the addition of an external regressor (presented in Section 5.8) or through AI methods (Section 5.9).

This model performs slightly worse than the standard ARIMA model for physical fidelity, as it is more likely to return negative values or values above that capable of the element. It therefore scores low for physical fidelity. This model is understandable as the sum of the seasonal and trend components, with an ARIMA modelled remainder. Parameters for the ARIMA component are readily available, and a general understanding of underlying patterns in the data is obtainable, however it is somewhat convoluted. Interpretability therefore scores low to moderate. The computational time to fit models to each household varied between 0.18s and 20.91s, with the average over all households taking 2.54s.

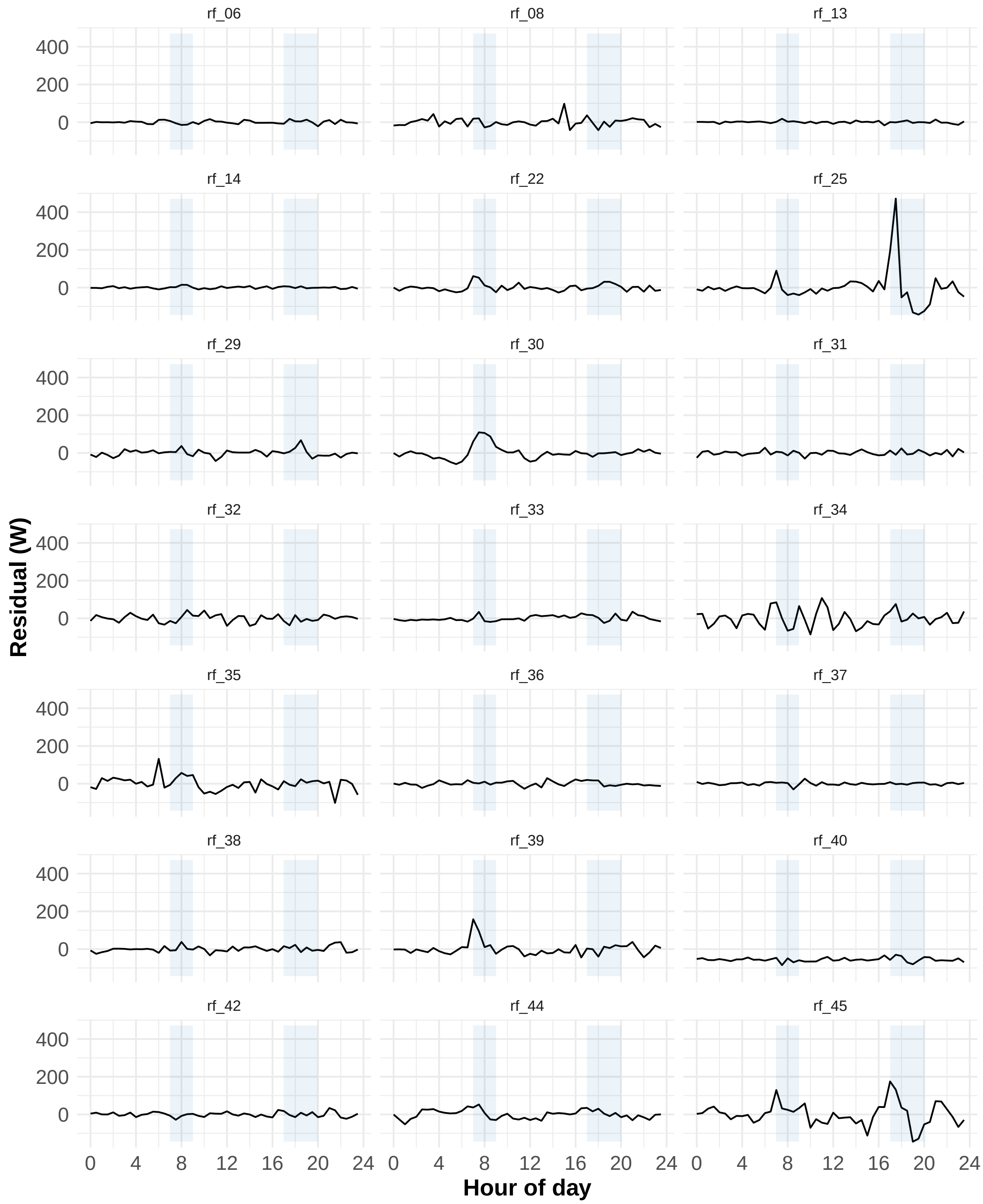

Figure 5.17: Daily profile of residuals for the STL + ARIMA model (peak periods shaded)

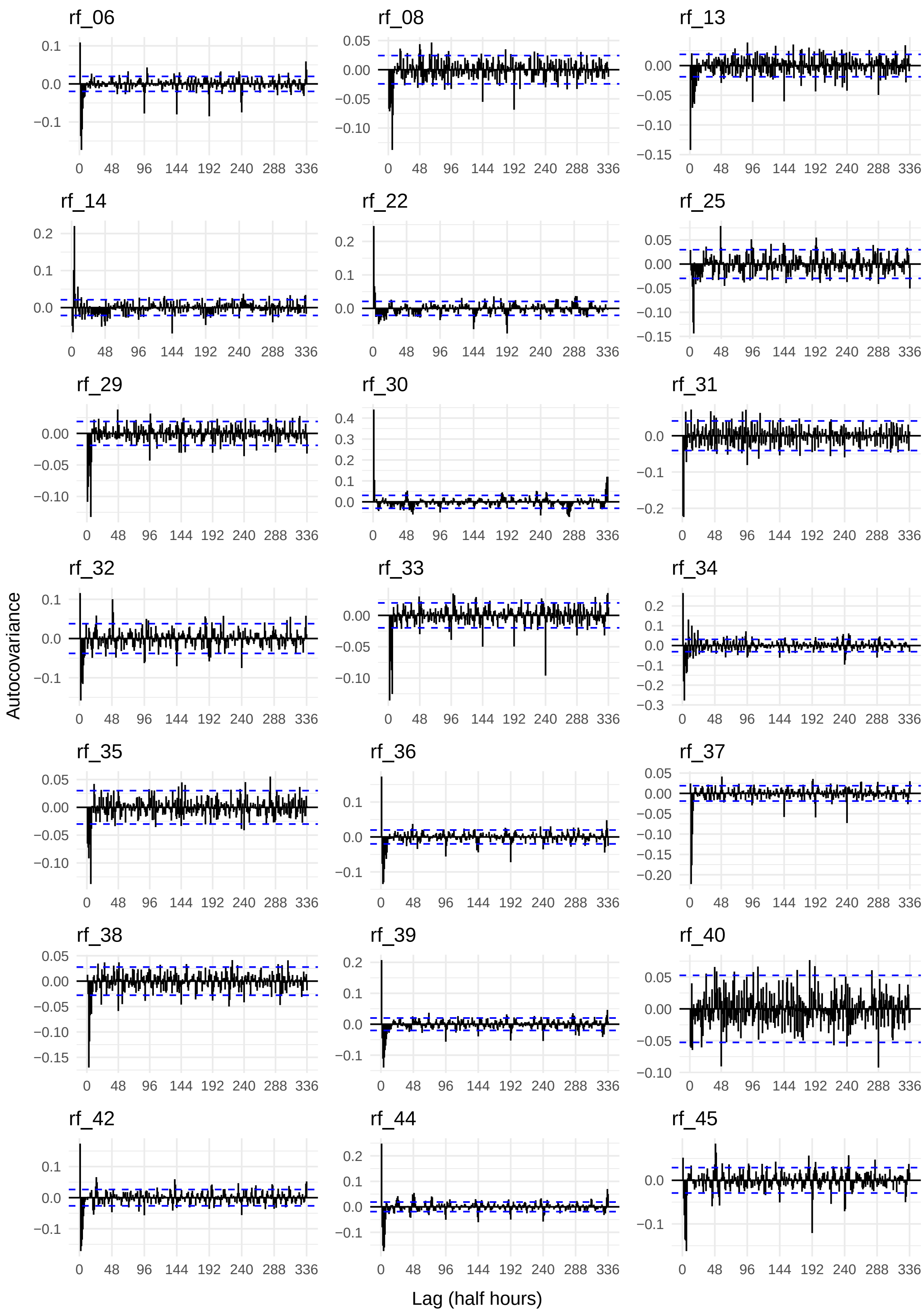

Figure 5.18: Autocovariance of residuals for the STL + ARIMA model

5.8 ARIMAX with STL decomposition

Figure 5.19: Performance of the STL + ARIMAX model for four households over four separate days

The most complex conventional forecasting method considered in this thesis was the STL + ARIMAX model outlined in Section 3.7.11. This model decomposes data using the STL method introduced in Section 3.4.2, then fits an ARIMAX model to the remainder component. As with the other models that incorporated regression, the previous two values of other appliances' electricity demand were selected as the regressor variables. Parameters for the ARIMA component of the model can be found in Table A.6. The performance of the model for four households is shown in Fig. 5.19.

Our STL + ARIMAX model had RMSEs ranging between 277W and 799W, with the average over all households being 508W. This corresponds to a 16% increase in accuracy compared to the naive model. During peak periods, RMSEs ranged between 292W and 1104W, with the average over all households being 637W. This corresponds to a peak period RMSE increase of 25.4%. This is slightly more than that of the same model without the external regressor (STL + ARIMA). Daily profiles of residuals are provided in Fig. 5.20. The autocovariance of residuals is also provided in Fig. 5.21. These are similar to those for the STL + ARIMAX model in Fig. 5.21, indicating that this model also captures underlying properties of the data reasonably well.

This model essentially performs the same as the ARIMA + STL model for physical fidelity. It can be quite effective in predicting initial instances of the element turning on, although it is also more likely to return negative values or values above that capable of the element. It therefore scores low for physical fidelity. This model is understandable as the sum of the seasonal and trend components, with an ARIMAX modelled remainder. Parameters for the ARIMAX component are readily available, and a general understanding of underlying patterns in the data is obtainable, however it is somewhat convoluted. Interpretability therefore scores low to moderate.
The computational time to fit models to each household varied between 0.76s and 34.66s, with the average over all households taking 5.38s.

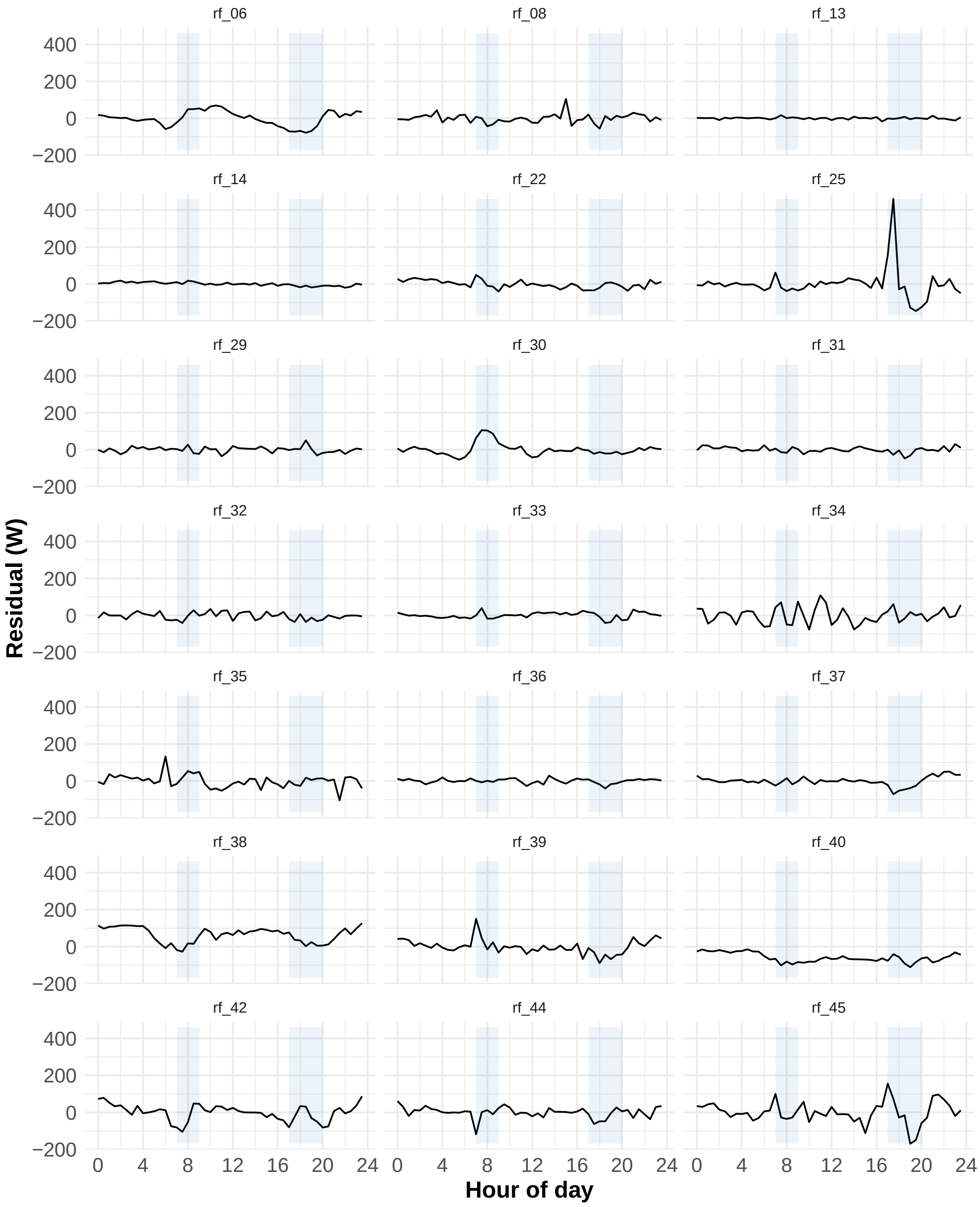

Figure 5.20: Daily profile of residuals for the STL + ARIMAX model (peak periods shaded)

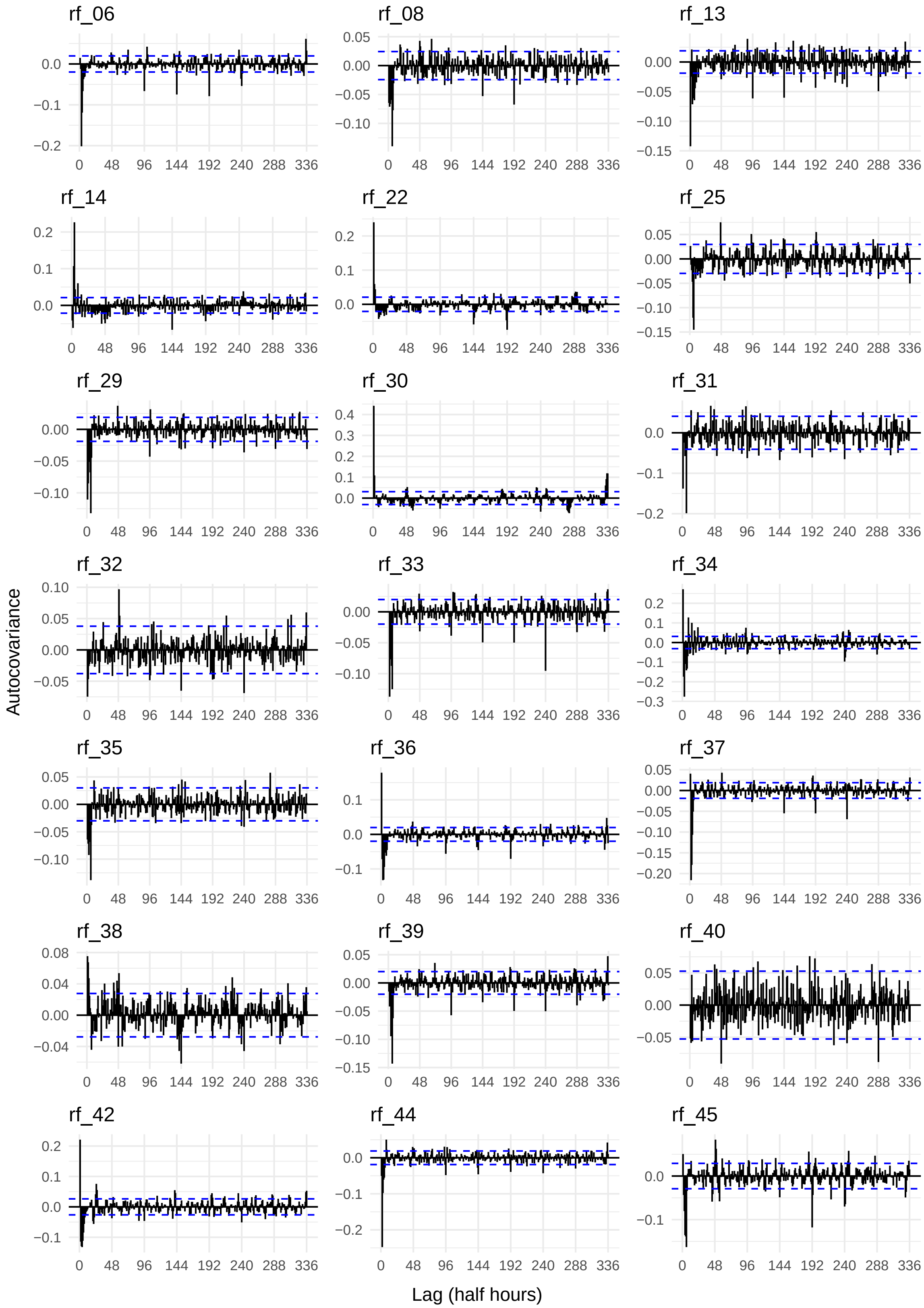

Figure 5.21: Autocovariance of residuals for the STL + ARIMAX model

5.9 SVM

The SVM model uses a complex machine learning algorithm in order to perform a linear regression on highly non-linear data. More details on this are provided in Section 3.7.12.

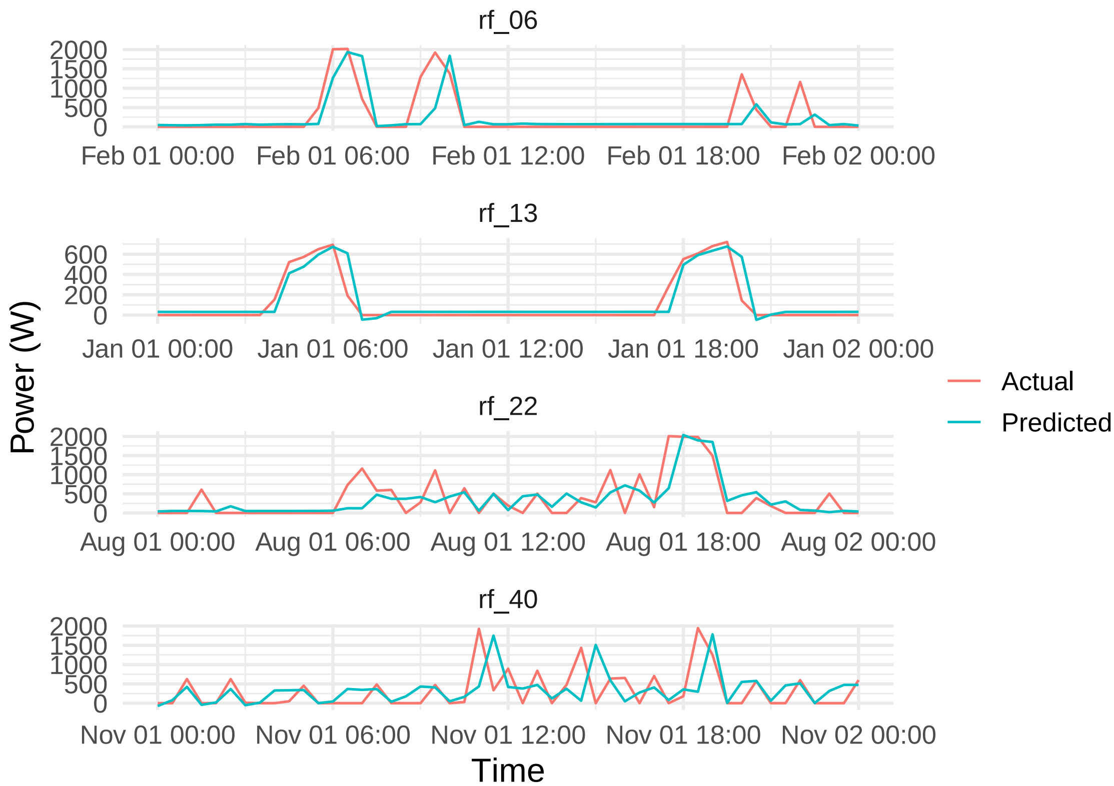

Figure 5.22: Performance of the SVM model for four households over four separate days

The timescale of initial decay in hot water electricity demand autocorrelation (refer to (3.7) and Section 4.2) occurred within a one hour timescale, thus the previous two values of x were provided as inputs into this model. Similarly, based on the results of cross-covariance (3.12) presented in Section 4.5, the previous two lagged values of y were also provided as inputs into this model. In addition, the hour of the day was provided as an input to account for seasonality. In terms of these variables, the classifying hyperplane determined in (3.27) becomes

$$\Gamma = \gamma_0 + \gamma_1 h_t + \gamma_2 x_{t-1} + \gamma_3 x_{t-2} + \gamma_4 y_{t-1} + \gamma_5 y_{t-2}. \tag{5.3}$$

The SVM was the most accurate model considered, with RMSEs ranging between 249W and 776W. The average over all households was 504W. This corresponds to a 16% increase in accuracy compared to the naive model. The performance of this model for four households can be seen in Fig. 5.22. During peak periods, RMSEs ranged between 321W and 1077W, with the average over all households being 671W. This model had the highest increase in peak period RMSEs of 33.3%, which is further indicated by the daily profile of residuals in Fig. 5.23. The autocovariance of residuals is also provided in Fig. 5.24. These indicate that the SVM model is capturing underlying properties of the data reasonably well.

The model is slightly smoother than the actual data, and seldom displays negative values or values above what is capable of the element. It scores highly for physical fidelity. This model is essentially 'black box', providing very limited insight as to its selected parameters and their significance. As such, it scores very low for interpretability. The computational time to fit models to each household was very high, varying between 2.51s and 393.2s, with the average over all households taking 112.12s.

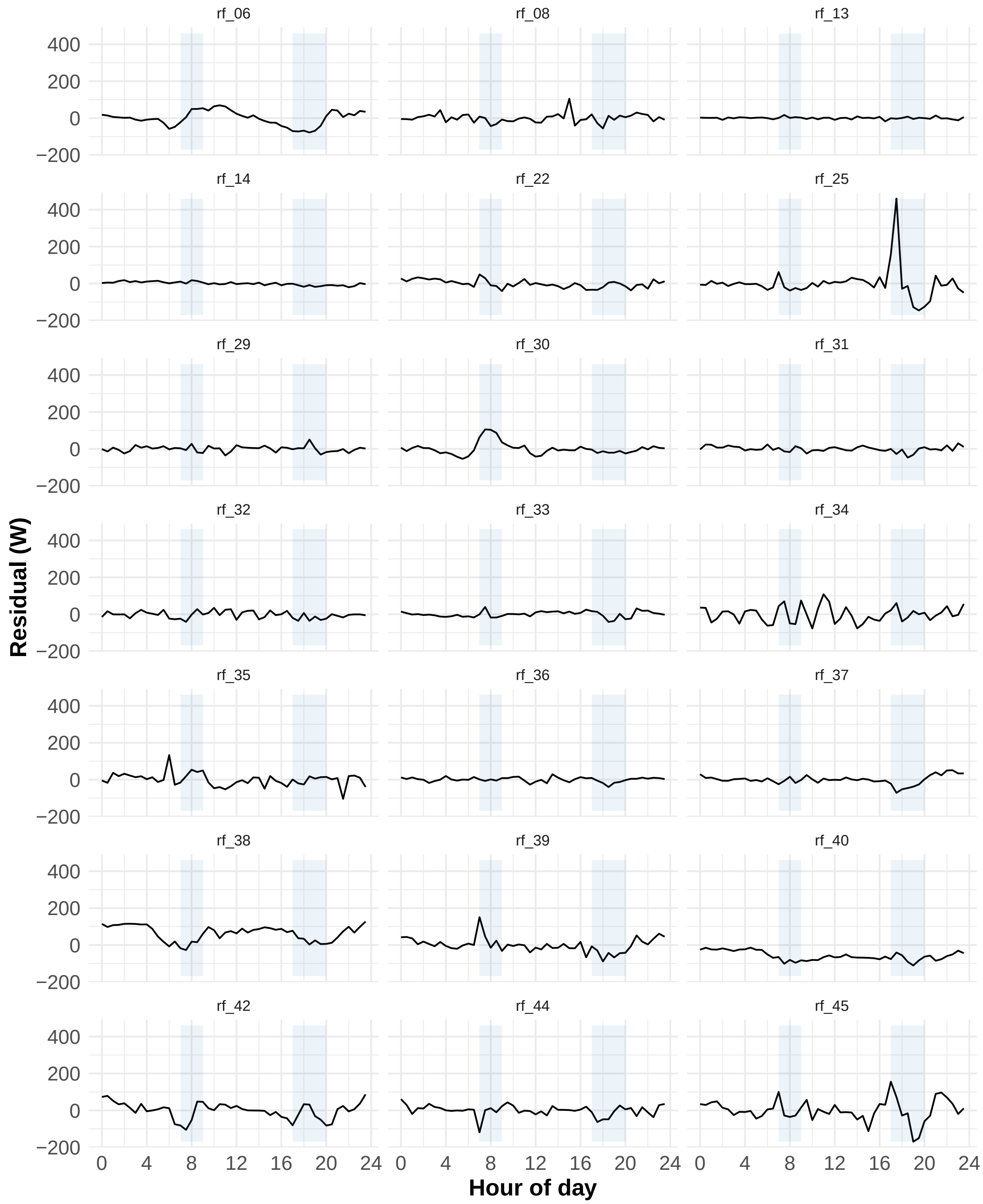

Figure 5.23: Daily profile of residuals for the SVM model (peak periods shaded)

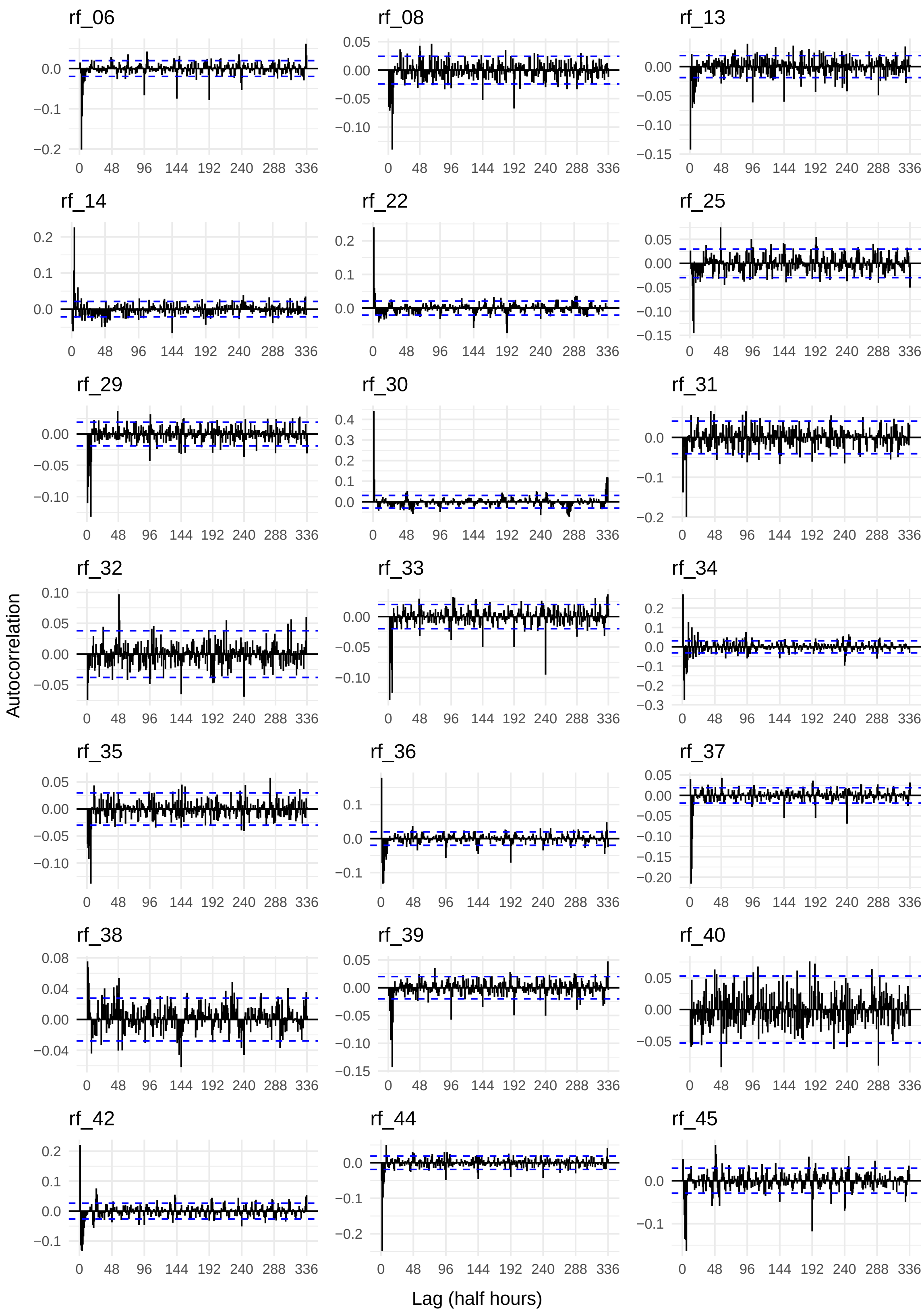

Figure 5.24: Autocovariance of residuals for the SVM model

Table 5.1: Average quantitative performance of each model

Model	RMSE (W)	peak period RMSE (W)	PEI (%)	Fitting time (s)
Seasonal naive	729.11	926.61	27.09	0.05
Simple linear	578.34	761.37	31.65	0.04
Naive	601.68	760.34	26.37	0.06
ARIMA	553.29	710.71	28.45	1.27
STL + ARIMA	514.49	643.65	25.10	2.54
ARIMAX	541.75	694.26	28.15	4.08
STL + ARIMAX	508.21	637.21	25.38	5.38
SVM	503.69	671.22	33.26	112.12

5.10 Summary

This chapter presented the qualitative and quantitative results of the models considered in this research. A summary of values for the quantitative metrics is provided in Table 5.1. When considering qualitative metrics, SVMs were rated highly for physical fidelity, but scored very low for interpretability. Linear regression and ARIMA models were given high and moderate ratings (respectively) for interpretability, but these models were lacking in physical fidelity.

Figures demonstrating the performance of four households were included to provide an intuition to the shape and general performance of each model. Daily profiles of model residuals were provided for each household. These highlight the performance of the model during peak periods, as these are the periods where smart control for demand response is most critical. Autocovariance plots of model residuals were also provided to demonstrate the ability of the model to capture seasonal and autoregressive effects. Following on from these results is Chapter 6, the discussion chapter, whereby these results are compared with one another in more detail, and the implications of these results to the aims of this thesis are elucidated.

Chapter 6

Discussion

The aim of this research was to compare methods of predicting household level hot water electricity demand using existing individual household electricity demand data. The ultimate purpose of household level hot water electricity demand prediction is to assist smart control objectives. In particular, demand predicting models may be used to improve demand response capacity, energy efficiency, and fidelity of hot water cylinder simulations. This chapter provides commentary on results presented in Chapter 5, in particular their performance against the metrics presented in Section 3.10. Implications on the broader objectives introduced in Chapter 1 are also presented, with a brief overview of how smart control with demand predictions may be incorporated into New Zealand's electricity grid.

6.1 Interpretations

In general, the most accurate model considered was SVMs (described in Section 3.7.12, results presented in Section 5.9). This wasn't surprising, as the suitability of SVMs for electricity demand forecasting was already noted in existing literature [56], [57], and was a large reason why they were selected as the comparative AI method. However, during peak times of day the SVM was outperformed in accuracy by models that incorporate STL decomposition. SVMs would therefore be recommended in a situation where accuracy of prediction during non-peak periods is the highest consideration. The SVM model also closely resembles the physical process, providing the most realistic 'looking' predictions of all (non-naive) models considered. However, when other metrics are taken into consideration, the SVM model falls short. The average number of seconds taken to fit the model to a household (112s) was at least two orders of magnitude higher than any other model considered. While 112s doesn't seem too long when considering a small amount of households, this may become prohibitive when applied to hundreds of thousands of households, even with high powered computing capacity. In addition, this model is has very low interpretability, and therefore would not be of much use in gaining deeper insights into behavioural patterns around domestic hot water use.

The inclusion of lagged values of other appliance electricity demand as a regressor to improve predictions of hot water electricity demand appears to be unique to this thesis, and may be considered an integral result. While linear regression of lagged values of other appliances was a relatively ineffective forecasting model on its own (refer to Section 5.3), it provided an accuracy boost to models that were already relatively accurate, such as the STL + ARIMA model (described in Section 3.7.9, results presented in Section 5.7) which was highly regarded in [26]. The STL + ARIMAX (described in Section 3.7.11, results presented in Section 5.8) built upon the STL + ARIMA model by adding lagged values of other appliance electricity demand as an external regressor. This addition provided it with accuracy almost rivalling SVMs, and surpassing SVMs during peak periods. In terms of additional accuracy improvements over the STL + ARIMA model, the external regressor improves RMSE by 6.3W on average, and by 6.4W during grid peaks. While the STL + ARIMAX had the slowest average computational speed of the conventional models, it was still vastly quicker than that of the SVM. The STL + ARIMAX model has many parameters, and their interpretation would require high familiarity with the model. This model would provide insights into underlying behavioural properties if properly interpreted, however simpler models would be better suited for this purpose. Where the STL + ARIMAX model really falls short is in its physical fidelity. Many values are predicted that are impossible for the element to achieve, and the model has a notably different shape to the underlying data.

As no single model scored highly across all comparative metrics, we must come to terms with trade-offs. Up to a point, increasing the complexity of a model tends to improve accuracy, while lowering interpretability and slowing computation. Eventually though, due to overfitting, adding additional parameters to a model will begin to lower accuracy while also still lowering performance in interpretation and computational speed. Trade-offs regarding the physical fidelity metric are less clear. While the physical fidelity of ARIMA based models decrease with increasing model complexity, the SVM model is both highly complex while still showing high physical fidelity.

6.2 Recommendations

While an ‘all-purpose’ model that scores highly in all metrics was not discovered, our results provide guidelines to choosing a suitable model for residential hot water electricity demand. The STL + ARIMAX model provided comparatively accurate predictions during peak periods, while remaining reasonably fast to compute, and thus would be the recommended model for most forecasting purposes. To the best of our knowledge this model has not been used within existing literature for electricity demand forecasting. The SVM provides slightly more accurate predictions outside of peak periods at the cost of significantly slower model fitting. When determining statistical properties of data for simulating demand or other research purposes, it would likely be best to utilise parameters from STL and ARIMA models separately, along with results from our preliminary data analysis in Chapter 4.

6.3 Stakeholders in smart control

Smart control of hot water cylinders has the ability to facilitate a number of benefits within an electricity network. By better matching demand to meet supply, we may reduce or defer grid infrastructure expenditure. By heating the tank when electricity spot prices are comparatively low, arbitrage profits are realisable. An aggregate of hot water cylinders under smart control may bid to sell their interruptible load in instantaneous response markets, or be remunerated for shedding load during congestion periods. Finally, energy efficiency of the cylinder itself can be increased.

Distribution of electricity in New Zealand can be thought of as a stream, running from generators at the top, through the systems operator, then EDBs, retailers, and finally the end consumer. To borrow an economics term, we refer to these separate entities as grid 'actors'. Technically, through the use of a separately metered hot water cylinder on a smart meter, any one of these actors could perform smart control. However, the financial benefits of smart control are not realised by every grid actor equally. This is an important consideration when considering where in the electricity 'stream' from generator to consumer would be the best place to position the control ability in order to maximise the realisation of these benefits.

Under New Zealand's current electricity market structure, we identify the key actors that stand to *directly* profit from these smart control benefits. Firstly, generators are only directly affected by spot price arbitrage. A shift in demand from high-cost electricity to low-cost electricity has the effect of smoothing price peaks. This benefits generators that provide non-peak electricity, while disadvantaging peak-price generators. The systems operator is in charge of transmitting electricity from generators to local substations, and can only directly gain from a reduction and deferral of infrastructure investment. As generators and the systems operator have comparatively little to gain from smart control of hot water cylinders, the controller would be better positioned in a location further downstream.

Electricity distribution businesses (EDBs), retailers and residential consumers all have the potential to realise multiple smart control benefits. A controller positioned within the residential household could provide energy efficiency gains and, if the household was exposed to spot pricing, electricity price arbitrage. However, one cylinder element could not offer enough demand shifting capacity to participate in IR markets, and residential consumers are not currently offered the ability to directly benefit from load reduction during congestion periods. Through controlling an aggregation of hot water cylinders, retailers have the ability to participate in all three markets, spot, IR, and CPD, however they would not directly benefit from better matching demand to supply, or increasing energy efficiency of the cylinder.

Finally, we consider the EDBs. EDBs are not exposed to spot market prices, and cannot benefit from spot price arbitrage. However, if they were controlling an aggregate of hot water cylinders, they would have the ability to directly profit from all other smart control benefits. Bids for interruptible load may be submitted into IR markets. Reducing CPD and increasing the energy efficiency of thousands of hot water cylinders within their network are considered direct benefits as at EDB level they would reduce and/or defer capital expenditure within the network.

Table 6.1 provides an overview of the benefits of smart control, and who may realise each of them.

Table 6.1: Benefits facilitated by smart control of hot water cylinders, and actors within the current New Zealand electricity market structure that may directly profit from each benefit

	Generators	Systems operator	EDBs	Retailers	Residential consumer
Grid efficiency	False	True	True	False	False
Spot price arbitrage	Uncertain	False	False	True	True
IR markets	False	False	True	True	False
CPD response	False	False	True	True	False
Energy efficiency	False	False	True	False	True

In addition to realising the most benefits, EDBs have advantages when physically establishing a smart control system. As most EDBs have already been controlling hot water cylinders through ripple control, they may have existing infrastructure they can repurpose for smart control.

6.4 Applying demand forecasting to smart control

This section provides a brief overview of how this research may be incorporated into a smart control system using technologies currently in existence in New Zealand. In the context of aggregating interruptible load to monetise demand shifting potential, the entity who performs the data aggregation and control is referred to as the 'aggregator'. We thus use this terminology for the remainder of the discussion. First, it must be noted that to maximise demand-shifting capabilities while minimising negative impacts on service, predictions of the electricity demand of the hot water cylinder is not sufficient. What is necessary is an accurate prediction of whether there will be sufficient hot water in the tank to cover demand. A simple physical model would have the ability of estimating and forecasting the volume of hot water in each tank, using assumptions about the tank size and water temperature along with measured and forecasted hot water electricity demand. It is for this reason that much emphasis has been given to the ability of prediction models to be incorporated into physical models.

The smart control system envisaged in this thesis is shown schematically in Fig. 6.1. For the system we propose, the smart meter from each household in the system would send the aggregator household level demand data, both from the separately metered hot water cylinder, and from other appliances. A forecasting model (such as the STL + ARIMAX developed in this thesis) would be used in order to predict future hot water electricity demand. This would be sent to a simple physical model, that estimates the volume of hot water in the cylinder based on historical electricity demand, and predicts what the future volume would be if electricity to the element was suppressed over the next half hour. If the future predicted hot water volume under this half-hour element suppression approaches zero, the HWC is left alone to operate without control. If the future predicted hot water volume under this half-hour element suppression doesn't become close to zero, the predicted electricity demand of this cylinder is then sent forward, along with all other suitable cylinders in this system, into a control algorithm. This technique ensures that the maximum capacity for demand shifting is attained. Element suppression scenarios for the next half hour are determined by cost-optimising participation in the various electricity and demand response markets. Once this control scenario is determined, bids into relevant markets can be made, and a signal is sent to the smart meters of participating households to suppress the elements of the hot water cylinders accordingly. Suppression of the element according to this scenario would automatically reduce the average temperature of water in the tank, improving energy efficiency.

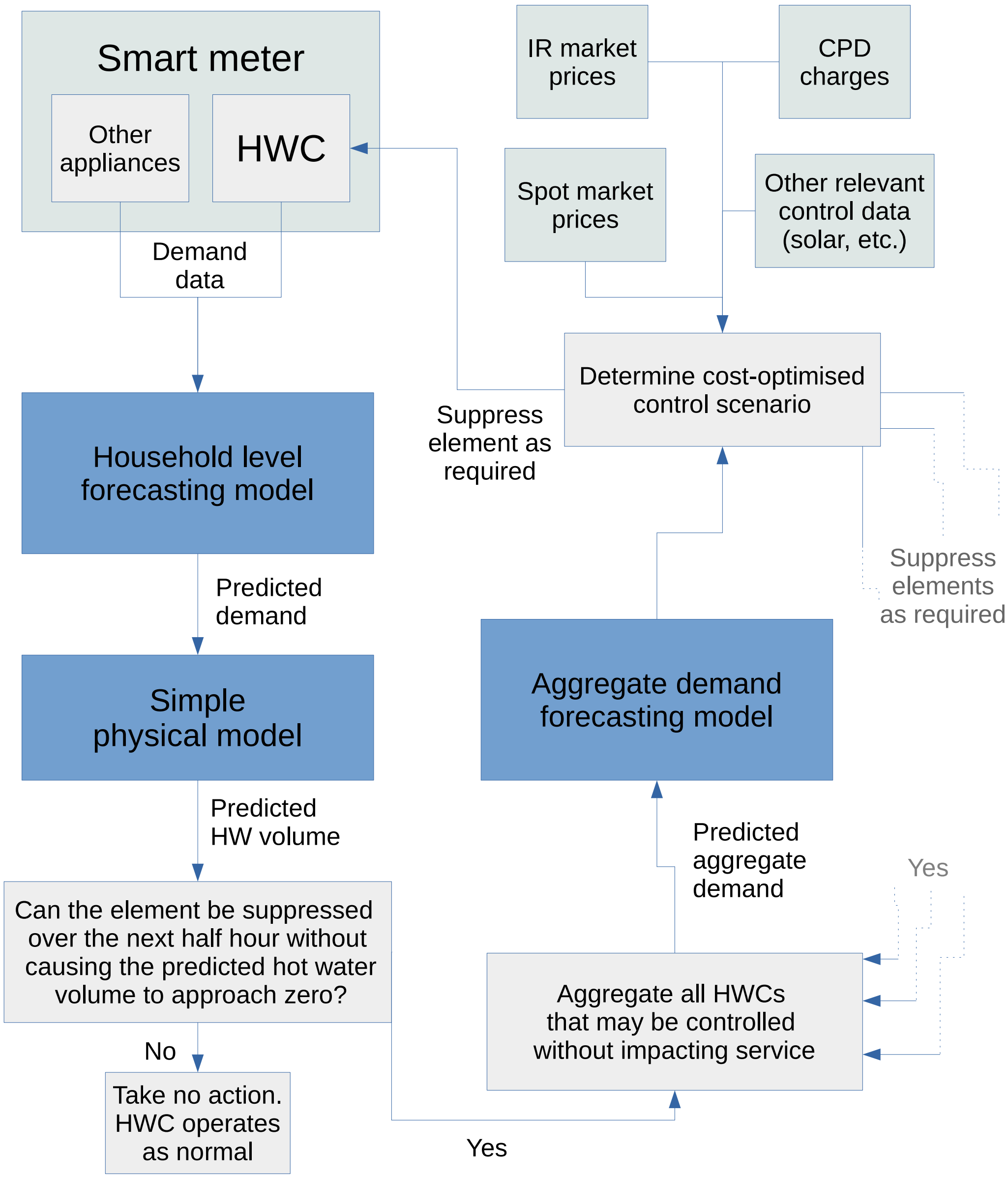

Figure 6.1: Schematic of smart control system implementing demand forecasting

6.5 Financial value

Under current New Zealand electricity market structures, a rough estimate can be made of overall income/savings available from this system. Drawing on prior research of the economic potential of heating domestic hot water cylinders only during prescribed times of day [5], annual savings around $40 per household per year are gained from spot price arbitrage, and an additional \$75 per household per year in reduced congestion period (CPD) charges. Prior research undertaken as part of a PGDipSci dissertation [105] indicated that the economic potential for hot water cylinders participating in New Zealand's fast instantaneous response market is approximately \$9 annually per household. This only assumes participation half of the time in alternating 15 minute periods. Taking the estimate of \$40 from spot price arbitrage, \$75 from CPD reduction, and \$9 from the FIR market, we get an annual value of approximately \$124 for each household. As far as we are aware, no research has been done regarding the economic potential of hot water cylinders participating in New Zealand's sustained instantaneous reserves markets, so we exclude this from the analysis. This analysis also does not include cost savings due to increased energy efficiency, which we may expect to be approximately 12% based on existing literature [23], [24]. Additional value could also be obtained for households with solar PV by incorporating solar data in the control algorithm to maximise solar self-consumption.

This research was conducted under the assumption that a more economically efficient electricity grid suppresses the price of electricity, which in turn accelerates electrification of high-carbon industries. While not explicitly discussed in this research, reductions in carbon dioxide could be achieved more directly by smart control which optimises use of renewable energy sources. Data relating to availability of renewable sources can be included as "other relevant control data" in the control system outlined in Fig. 6.1.

Chapter 7

Conclusion

Smart control of hot water cylinders has a role to play in a smooth and cost-effective transition to a low-carbon society. To best realise this potential, we need the ability to make accurate demand forecasts at the level of individual cylinders.

This thesis set out to build and compare a selection of models to forecast residential hot water electricity demand using only data in the format currently available from smart meters in New Zealand. An underlying goal was to recommend an 'optimal' hot water electricity demand forecasting model for incorporation into a smart-control system. Optimal models for this purpose should provide accurate results at household level, and be reasonably fast to compute. Models were also compared according to their suitability for incorporation in physical simulations. This consideration required model outputs to be interpretable, and provide outputs that were reasonably similar to the original data.

The results suggest the STL + ARIMAX model shows the most promise for incorporation within a smart control system. This model involves decomposition of the hot water demand data into seasonal, trend and remainder components, and modelling the remainder as an ARIMAX process with lagged values of other electricity demand as an external variable. This method provided a combination of accuracy, computational speed and interpretability that was unmatched in other methods considered. It was, however, outperformed by a number of other methods in physical fidelity. This was due to the output producing results that were not physically obtainable by the cylinder element, as well as having higher stochasticity than the data.

7.1 Potential for further work

The research conducted for this thesis has provided groundwork for additional research relating to both smart control, and appliance level demand forecasting. This additional research may broadly be separated into two categories; (i) better quantifying the benefits of incorporating forecasting within smart control, and (ii) further refinements of the models, including adaptation and evaluation against different data formats.

7.1.1 Quantifying the benefits of forecasting within smart control objectives

The smart control system proposed in Section 6.4 has a number of benefits over New Zealand's existing ripple control system. These benefits include:

- accuracy improvements of predicted interruptible load
- energy efficiency improvements
- reduced impacts on service
- additional capacity for control

The obvious next step for this work is to incorporate the recommended forecasting model into hot water cylinder smart control scenarios (either simulated or real world) and determine the additional benefits it can provide over control methods that do not incorporate demand forecasting. As part of this process, work must be done in translating parameters of the forecasting models into demand simulation parameters, as part of the physical model mentioned in Fig. 6.1.

New Zealand's current ripple control system suppresses an aggregate of elements equally, with no consideration to individualised use patterns. In order to reduce impacts on service, the aggregate of elements can only be suppressed infrequently for short lengths of time, so that higher demand households do not run out of hot water. The necessity of excluding households that may be negatively impacted by control means that all other households have more capacity for control than is currently undertaken. While this is a reasonable assumption, the extent to which smart control with demand forecasting offers additional control capacity has not been quantified within this research. Further work could be made to estimate the quantity of this additional capacity. In addition, energy efficiency improvements gained by controlling according to the system presented in Fig. 6.1 could be quantified.

7.1.2 Additional model refining and testing

Some of the models presented in this thesis have the ability to be further refined to improve predictions. The SVM model in particular provided inferior results during peak periods, which are the times when demand response is most valuable. Peak period forecast accuracy of this model could potentially be improved by including additional parameters.

Another research angle would be to refine the models in this research to forecast higher resolution data. The element within a typical hot water cylinder is controlled thermostatically by an "on-off" (otherwise known as "bang-bang") controller. This type of controller turns the element fully "on" when the water temperature drops below a certain temperature, and then fully off again once the water reaches another (slightly higher) temperature. On small timescales, the element may then be considered a binary process, impacting our choice of forecasting model. Forecasting a binary output would involve the addition of a logistic function [103] to existing models. Rather than producing a continuous output of the predicted element power as used in this thesis, models that incorporate a logistic function produce an output of the probability the element is on. This accommodates binary (on/off) predictions of element state.

An 'aggregate demand forecasting model' is mentioned in Fig. 6.1. This refers to a forecast of aggregate demand for the households able to participate in demand response. Forecasting models developed in [26] showed that when fitted to aggregated demand data, accuracy of predictions were higher than when fitted to individual household data. It is unclear whether a sum of household level forecasts would outperform a forecast fitted to aggregated demand data using the recommended models in this thesis due to the use of other appliance electricity demand as an external regressor. The higher-performing models in this thesis could be fitted and tested using aggregated demand data to ascertain this.

To reduce computational time and model complexity, only half-hour and one-hour lags of other appliance demand were used as external regressors in models for all selected households. It can be seen in Table A.2 in the Appendix and the corresponding Fig. 4.5 in Chapter 3 that some households have optimal cross-covariance lags that are greater than one hour. Household-specific lag values of external regressors could be incorporated into models as further work.

There is also potential for further exploration of the models considered in this work for forecasting the demand of other deferrable loads, such as electric vehicles.

7.2 Closing remarks

This thesis compares models for forecasting individual household level hot water electricity demand, and proposes a particular smart control system that incorporates them. Smart control of hot water cylinders has the potential to increase both the economic efficiency of an electricity grid, and the energy efficiency of domestic hot water use. Both of these efficiency gains suppress the price of electricity, hastening the electrification of high-carbon industries such as transport and industrial processing. Electricity demand forecasting provides a demonstrable improvement to smart control systems, and as such, is an important tool in the transition to a low-carbon society.

Appendix A

Tables and parameters

A.1 Seasonality

Table A.1: Three dominant cycle periods for hot water electricity demand of each household, as determined using frequency analysis

Household	Dominance	Period (hours)
rf_06	1	11.99
rf_06	2	11.96
rf_06	3	23.94
rf_08	1	8192.00
rf_08	2	12.01
rf_08	3	11.99
rf_13	1	23.98
rf_13	2	11.99
rf_13	3	27000.00
rf_14	1	11.99
rf_14	2	11.99
rf_14	3	11.98
rf_22	1	22118.40
rf_22	2	23.96
rf_22	3	11059.20
rf_25	1	12.00
rf_25	2	23.90
rf_25	3	4.80
rf_29	1	23.98
rf_29	2	11.98
rf_29	3	11.98
rf_30	1	24.00
rf_30	2	12.00
rf_30	3	8.00

rf_31	1	12.00
rf_31	2	12.02
rf_31	3	12.05
rf_32	1	11.99
rf_32	2	24.02
rf_32	3	5.99
rf_33	1	12.00
rf_33	2	24.00
rf_33	3	11664.00
rf_34	1	23.98
rf_34	2	11.99
rf_34	3	10000.00
rf_35	1	11.99
rf_35	2	24.02
rf_35	3	23.78
rf_36	1	12.00
rf_36	2	12.01
rf_36	3	11.99
rf_37	1	11.99
rf_37	2	12.00
rf_37	3	12.00
rf_38	1	12.00
rf_38	2	24.03
rf_38	3	6.00
rf_39	1	11.99
rf_39	2	12.00
rf_39	3	23.97
rf_40	1	11.98
rf_40	2	23.97
rf_40	3	5.99
rf_42	1	11.99
rf_42	2	12.00
rf_42	3	11.97
rf_44	1	11.99
rf_44	2	6.00
rf_44	3	12.00
rf_45	1	12.00
rf_45	2	24.00
rf_45	3	11.98

A.2 Cross-covariance lag

Table A.2: Lag time and correlation value at point of maximum cross-covariance

Household	Lag	Cross covariance
rf_01	895	0.1330911
rf_02	30	0.1934237
rf_06	30	0.2550428
rf_08	30	0.1406060
rf_11	756	0.0214757
rf_12	30	0.0737116
rf_13	216	0.1831276
rf_14	30	0.1199669
rf_16	681	0.0712576
rf_17a	812	0.0745542
rf_18	30	0.3384360
rf_20	30	0.0904845
rf_22	30	0.1358168
rf_23	59	0.1191495
rf_24	505	0.2144240
rf_25	69	0.0821539
rf_27	111	0.1206740
rf_29	73	0.1023941
rf_30	818	0.1893829
rf_31	57	0.0121780
rf_32	76	0.1580285
rf_33	30	0.1148974
rf_34	921	0.1673047
rf_35	752	0.1289042
rf_36	30	0.1450114
rf_37	30	0.1906087
rf_38	80	0.1906016
rf_39	758	0.1319346
rf_40	30	0.1598445
rf_42	30	0.2798608
rf_44	30	0.2963463
rf_45	1428	0.1623588

A.3 Modelling parameters

Table A.3 show the ARIMA parameters that minimise the AIC as selected by the `auto.arima` function. As described in Chapter 3, p refers to the order of autogression, d refers to the order of differencing necessary to obtain stationarity, and q is the order of moving average. Similarly, Tables A.4, A.5 and A.6 show the parameters for the ARIMAX, STL + ARIMA, and STL + ARIMAX models respectively.

Table A.3: Optimal parameters for the ARIMA models

household	p	d	q
rf_06	0	1	2
rf_08	5	1	0
rf_13	0	1	4
rf_14	0	1	3
rf_22	0	1	1
rf_25	0	1	1
rf_29	0	1	1
rf_30	0	1	1
rf_31	1	1	0
rf_32	0	1	1
rf_33	0	1	1
rf_34	0	1	2
rf_35	0	1	1
rf_36	0	1	1
rf_37	0	1	1
rf_38	0	1	2
rf_39	0	1	1
rf_40	1	0	1
rf_42	0	1	1
rf_44	0	1	1
rf_45	5	1	0

Table A.4: Optimal parameters for the ARIMAX models

household	p	d	q
rf_06	0	1	2
rf_08	5	1	0
rf_13	0	1	4
rf_14	0	1	3
rf_22	0	1	1
rf_25	5	1	0
rf_29	0	1	1
rf_30	0	1	1
rf_31	1	0	2
rf_32	2	0	3
rf_33	5	1	0
rf_34	0	1	2
rf_35	0	1	1
rf_36	1	0	1
rf_37	0	1	1
rf_38	0	1	2
rf_39	0	1	1
rf_40	1	0	3
rf_42	0	1	1
rf_44	0	1	1
rf_45	5	1	0

Table A.5: Optimal parameters for the STL + ARIMA models

household	p	d	q
rf_06	0	1	1
rf_08	5	1	0
rf_13	0	1	3
rf_14	1	1	1
rf_22	0	1	1
rf_25	5	1	0
rf_29	5	1	0
rf_30	0	1	1
rf_31	1	1	0
rf_32	0	1	1
rf_33	5	1	0
rf_34	0	1	1
rf_35	5	1	0
rf_36	0	1	1
rf_37	0	1	1
rf_38	2	1	0
rf_39	0	1	1
rf_40	1	0	1
rf_42	0	1	1
rf_44	0	1	1
rf_45	5	1	0

Table A.6: Optimal parameters for the STL + ARIMAX models

household	p	d	q
rf_06	2	1	0
rf_08	5	1	0
rf_13	0	1	3
rf_14	1	1	1
rf_22	0	1	1
rf_25	5	1	0
rf_29	5	1	0
rf_30	0	1	1
rf_31	5	1	0
rf_32	5	0	2
rf_33	5	1	0
rf_34	0	1	1
rf_35	5	1	0
rf_36	0	1	1
rf_37	0	1	1
rf_38	1	0	3
rf_39	5	1	0
rf_40	1	0	1
rf_42	0	1	1
rf_44	1	1	0
rf_45	5	1	0

References

[1] R. K. Pachauri *et al.*, "Climate Change 2014: Synthesis Report. Contribution of Working Groups I, II and III to the Fifth Assessment Report of the Intergovernmental Panel on Climate Change," 2014.

[2] J. H. Williams *et al.*, "2050 : The Pivotal Role of Electricity," *Science (New York, N.Y.)*, vol. 335, pp. 53–60, 2012.

[3] B. Anderson *et al.*, "New Zealand GREEN Grid Household Electricity Demand Study 2014-2018," *UK Data Service*, 2018.

[4] J. Stephenson, R. Ford, N. K. Nair, N. Watson, A. Wood, and A. Miller, "Smart grid research in New Zealand – A review from the GREEN Grid research programme," *Renewable and Sustainable Energy Reviews*, vol. 82, pp. 1636–1645, 2018 [Online]. Available: `https://doi.org/10.1016/j.rser.2017.07.010`

[5] C. Dortans, B. Anderson, J. Stephenson, and M. Jack, "Estimating the Technical Potential of Residential Demand Response in New Zealand : A Summary of Results," *GREEN Grid Project Report*, pp. 1–12, 2018.

[6] M. Jack, K. Suomalainen, J. Dew, and D. Eyers, "Active and Passive Energy Storage Technologies GREEN Grid project : 2012 - 2018," *EMANZ Annual Conference*, 2017.

[7] M. Jack, R. Ford, J. Dew, M. Mason, and H. Pietsch, "Opportunities for demand management in the smart grid," *EMANZ Annual Conference*, 2016.

[8] P. D. Lund, J. Lindgren, J. Mikkola, and J. Salpakari, "Review of energy system flexibility measures to enable high levels of variable renewable electricity," *Renewable and Sustainable Energy Reviews*, vol. 45, pp. 785–807, 2015 [Online]. Available: `http://dx.doi.org/10.1016/j.rser.2015.01.057`

[9] G. Strbac, "Demand side management: Benefits and challenges," *Energy Policy*, vol. 36, no. 12, pp. 4419–4426, 2008.

[10] Transpower New Zealand, "Transmission Planning Report," 2015 [Online]. Available: `https://www.transpower.co.nz/resources/transmission-planning-report-2015`

[11] Electricity Authority New Zealand, "Electricity in New Zealand," 2018

[Online]. Available: https://www.ea.govt.nz/about-us/media-and-publications/electricity-new-zealand/

[12] Electricity Authority New Zealand, "Enabling a national market for instantaneous reserve," 2014 [Online]. Available: https://www.ea.govt.nz/dmsdocument/18797

[13] Orion, "How to use ripple signals on Orion's network," 2017 [Online]. Available: https://www.oriongroup.co.nz/assets/Customers/RippleSignalGuide.pdf

[14] N. K. C. Nair, R. Nayagam, R. Francis, A. Transmission, and W. Market, "New Zealand Utility Experiences with Demand Side Management," *IEEE Power and Energy Society General Meeting, Pittsburgh, PA, USA*, pp. 1–5, 2008.

[15] New Zealand Electricity Commission, "Advanced Metering Infrastructure in New Zealand: Roll-out and Requirements," 2009 [Online]. Available: https://www.ea.govt.nz/operations/retail/metering/advanced-metering/

[16] R. Lawson, P. Thorsnes, and J. Williams, "Consumer Response to Time Varying Prices for Electricity," 2011 [Online]. Available: https://www.otago.ac.nz/economics/research/otago076674.pdf

[17] S. Okajima and H. Okajima, "Estimation of Japanese price elasticities of residential electricity demand, 1990-2007," *Energy Economics*, vol. 40, pp. 433–440, 2013.

[18] M. Filippini, "Short- and long-run time-of-use price elasticities in Swiss residential electricity demand," *Energy Policy*, vol. 39, pp. 5811–5817, 2011.

[19] K. Spiliotis, A. I. Ramos Gutierrez, and R. Belmans, "Demand flexibility versus physical network expansions in distribution grids," *Applied Energy*, vol. 182, pp. 613–624, 2016 [Online]. Available: http://dx.doi.org/10.1016/j.apenergy.2016.08.145

[20] R. D'hulst, W. Labeeuw, B. Beusen, S. Claessens, G. Deconinck, and K. Vanthournout, "Demand response flexibility and flexibility potential of residential smart appliances: Experiences from large pilot test in Belgium," *Applied Energy*, vol. 155, pp. 79–90, 2015 [Online]. Available: http://dx.doi.org/10.1016/j.apenergy.2015.05.101

[21] New Zealand Electricity Commission, "New Zealand Electric Energy-Efficiency Potential Study Volume 1," vol. 1, 2007 [Online]. Available: https://www.eeca.govt.nz/assets/Resources-EECA/New-Zealand-Electric-Energy-Efficiency-Potential-Study-main-report.pdf

[22] A. Pollard, A. Stoecklein, M. Camilleri, L. Amitrano, and N. Isaacs, "An Initial Investigation into New Zealand's Residential Hot Water Energy Usage," *IRHACE Technical Conference*, vol. 99, no. 99, 2002.

[23] P. Kepplinger, G. Huber, and J. Petrasch, "Autonomous optimal control for

demand side management with resistive domestic hot water heaters using linear optimization," *Energy and Buildings*, vol. 100, pp. 50–55, 2015 [Online]. Available: http://dx.doi.org/10.1016/j.enbuild.2014.12.016

[24] Y. Denis, F. Suard, A. Lomet, and D. Chèze, "Saving energy by anticipating hot water production: Identification of key points for an efficient statistical model integration," *Artificial Intelligence for Engineering Design, Analysis and Manufacturing: AIEDAM*, vol. 33, no. 2, pp. 138–147, 2019.

[25] A. Lomet, F. Suard, and D. Chèze, "Statistical modeling for real domestic hot water consumption forecasting," *Energy Procedia*, vol. 70, pp. 379–387, 2015 [Online]. Available: http://dx.doi.org/10.1016/j.egypro.2015.02.138

[26] L. Gelažanskas and K. A. A. Gamage, "Forecasting Hot Water Consumption in Residential Houses," *Energies*, vol. 8, pp. 12702–12717, 2015.

[27] M. A. Mat Daut, M. Y. Hassan, H. Abdullah, H. A. Rahman, M. P. Abdullah, and F. Hussin, "Building electrical energy consumption forecasting analysis using conventional and artificial intelligence methods: A review," *Renewable and Sustainable Energy Reviews*, vol. 70, pp. 1108–1118, 2016 [Online]. Available: http://dx.doi.org/10.1016/j.rser.2016.12.015

[28] M. W. Jack, K. Suomalainen, J. J. W. Dew, and D. Eyers, "A minimal simulation of the electricity demand of a domestic hot water cylinder for smart control," *Applied Energy*, vol. 211, pp. 104–112, 2018.

[29] M. Roux, M. Apperley, and M. J. Booysen, "Comfort, peak load and energy: Centralised control of water heaters for demand-driven prioritisation," *Energy for Sustainable Development*, vol. 44, pp. 78–86, 2018 [Online]. Available: https://doi.org/10.1016/j.esd.2018.03.006

[30] M. Roux, N. H. Naude, M. J. Booysen, and A. Barnard, "Electric water heaters in smartgrids: Individual savings versus network peak load management," in *SAUPEC, Stellenbosch*, 2017.

[31] A. Sepulveda, L. Paull, W. G. Morsi, H. Li, C. P. Diduch, and L. Chang, "A novel demand side management program using water heaters and particle swarm optimization," in *EPEC 2010 - IEEE electrical power and energy conference: "Sustainable energy for an intelligent grid"*, 2010.

[32] L. Gelažanskas and K. A. A. Gamage, "Distributed Energy Storage Using Residential Hot Water Heaters," *Energies*, vol. 9, no. 3, 2016.

[33] P. S. P. Cowpertwait and A. V. Metcalfe, *Introductory Time Series with R*. Springer, 2009 [Online]. Available: http://arxiv.org/abs/arXiv:1011.1669v3

[34] T. Hong and S. Fan, "Probabilistic electric load forecasting: A tutorial review," *International Journal of Forecasting*, vol. 32, no. 3, pp. 914–938, 2016 [Online]. Available: http://dx.doi.org/10.1016/j.ijforecast.2015.11.011

[35] Y. Wei *et al.*, "A review of data-driven approaches for prediction and classification of building energy consumption," *Renewable and Sustainable Energy Reviews*, vol. 82, pp. 1027–1047, 2018 [Online]. Available: http://dx.doi.org/10.1016/j.rser.2017.09.108

[36] N. Arghira, L. Hawarah, S. Ploix, and M. Jacomino, "Prediction of appliances energy use in smart homes," *Energy*, vol. 48, no. 1, pp. 128–134, 2012 [Online]. Available: http://dx.doi.org/10.1016/j.energy.2012.04.010

[37] D. Popescu and E. Serban, "Simulation of domestic hot-water consumption using time-series models," *6th IASME/WSEAS International Conference on heat transfer, thermal engineering and environment*, 2016.

[38] J. Kondoh, N. Lu, and D. J. Hammerstrom, "An evaluation of the water heater load potential for providing regulation service," *IEEE Transactions on Power Systems*, vol. 26, no. 3, pp. 1309–1316, 2011.

[39] A. H. Mohsenian-Rad, V. W. S. Wong, J. Jatskevich, R. Schober, and A. Leon-Garcia, "Autonomous demand-side management based on game-theoretic energy consumption scheduling for the future smart grid," *IEEE Transactions on Smart Grid*, vol. 1, no. 3, pp. 320–331, 2010.

[40] E. O'Shaughnessy, D. Cutler, K. Ardani, and R. Margolis, "Solar plus: A review of the end-user economics of solar PV integration with storage and load control in residential buildings," *Applied Energy*, vol. 228, pp. 2165–2175, 2018 [Online]. Available: https://doi.org/10.1016/j.apenergy.2018.07.048

[41] Q. Cui, X. Wang, X. Wang, and Y. Zhang, "Residential appliances direct load control in real-time using cooperative game," *IEEE Transactions on Power Systems*, vol. 31, no. 1, pp. 226–233, 2016.

[42] V. Kapsalis, G. Safouri, and L. Hadellis, "Cost/comfort-oriented optimization algorithm for operation scheduling of electric water heaters under dynamic pricing," *Journal of Cleaner Production*, vol. 198, pp. 1053–1065, 2018 [Online]. Available: https://doi.org/10.1016/j.jclepro.2018.07.024

[43] S. D. Ramchurn, P. Vytelingum, A. Rogers, and N. R. Jennings, "Agent-based homeostatic control for green energy in the smart grid," *ACM Transactions on Intelligent Systems and Technology*, vol. 2, no. 4, pp. 1–28, 2012.

[44] C. Passenberg, D. Meyer, J. Feldmaier, and H. Shen, "Optimal water heater control in smart home environments," *2016 IEEE International Energy Conference*, 2016 [Online]. Available: http://arxiv.org/abs/arXiv:1607.05587v1

[45] Z. Yu, L. Jia, M. C. Murphy-Hoye, A. Pratt, and L. Tong, "Modeling and stochastic control for home energy management," *IEEE Transactions on Smart Grid*, vol. 4, no. 4, pp. 2244–2255, 2013.

[46] M. De Felice, A. Alessandri, and P. M. Ruti, "Electricity demand forecasting

over Italy: Potential benefits using numerical weather prediction models," *Electric Power Systems Research*, vol. 104, pp. 71–79, 2013 [Online]. Available: http://dx.doi.org/10.1016/j.epsr.2013.06.004

[47] A. Jain and B. Satish, "Clustering based short term load forecasting using support vector machines," *2009 IEEE Bucharest PowerTech: Innovative Ideas Toward the Electrical Grid of the Future*, pp. 1–8, 2009.

[48] Y. Goude, R. Nedellec, and N. Kong, "Local short and middle term electricity load forecasting with semi-parametric additive models," *IEEE Transactions on Smart Grid*, vol. 5, no. 1, pp. 440–446, 2014.

[49] K. Gajowniczek and T. Zabkowski, "Short term electricity forecasting using individual smart meter data," *Procedia Computer Science*, vol. 35, pp. 589–597, 2014 [Online]. Available: http://dx.doi.org/10.1016/j.procs.2014.08.140

[50] A. S. Ahmad *et al.*, "A review on applications of ANN and SVM for building electrical energy consumption forecasting," *Renewable and Sustainable Energy Reviews*, vol. 33, pp. 102–109, 2014 [Online]. Available: http://dx.doi.org/10.1016/j.rser.2014.01.069

[51] F. McLoughlin, A. Duffy, and M. Conlon, "Evaluation of time series techniques to characterise domestic electricity demand," *Energy*, vol. 50, no. 1, pp. 120–130, 2013 [Online]. Available: http://dx.doi.org/10.1016/j.energy.2012.11.048

[52] H. Aki, T. Wakui, and R. Yokoyama, "Development of a domestic hot water demand prediction model based on a bottom-up approach for residential energy management systems," *Applied Thermal Engineering*, vol. 108, pp. 697–708, 2016 [Online]. Available: http://dx.doi.org/10.1016/j.applthermaleng.2016.07.094

[53] M. Molina-Solana, M. Ros, M. D. Ruiz, J. Gómez-Romero, and M. J. Martin-Bautista, "Data science for building energy management: A review," *Renewable and Sustainable Energy Reviews*, vol. 70, pp. 598–609, 2017 [Online]. Available: http://dx.doi.org/10.1016/j.rser.2016.11.132

[54] L. Wei, W. Tian, E. A. Silva, R. Choudhary, Q. Meng, and S. Yang, "Comparative Study on Machine Learning for Urban Building Energy Analysis," *Procedia Engineering*, vol. 121, pp. 285–292, 2015 [Online]. Available: http://dx.doi.org/10.1016/j.proeng.2015.08.1070

[55] F. McLoughlin, A. Duffy, and M. Conlon, "A clustering approach to domestic electricity load profile characterisation using smart metering data," *Applied Energy*, vol. 141, pp. 190–199, 2015 [Online]. Available: http://dx.doi.org/10.1016/j.apenergy.2014.12.039

[56] F. Kaytez, M. C. Taplamacioglu, E. Cam, and F. Hardalac, "Forecasting electricity consumption: A comparison of regression analysis, neural networks and least squares support vector machines," *International Journal of Electrical Power

and Energy Systems, vol. 67, pp. 431–438, 2015 [Online]. Available: http://dx.doi.org/10.1016/j.ijepes.2014.12.036

[57] J. Massana, C. Pous, L. Burgas, J. Melendez, and J. Colomer, "Short-term load forecasting in a non-residential building contrasting models and attributes," *Energy and Buildings*, vol. 92, pp. 322–330, 2015.

[58] N. Dervilis, K. Worden, T. Rogers, D. Scott, and T. Simpson, "Machine Learning for Energy Load Forecasting," *Journal of Physics: Conference Series*, vol. 1106, 2018.

[59] H. Chitsaz, H. Shaker, H. Zareipour, D. Wood, and N. Amjady, "Short-term electricity load forecasting of buildings in microgrids," *Energy and Buildings*, vol. 99, pp. 50–60, 2015 [Online]. Available: http://dx.doi.org/10.1016/j.enbuild.2015.04.011

[60] F. Zhang, C. Deb, S. E. Lee, J. Yang, and K. W. Shah, "Time series forecasting for building energy consumption using weighted Support Vector Regression with differential evolution optimization technique," *Energy and Buildings*, vol. 126, pp. 94–103, 2016 [Online]. Available: http://dx.doi.org/10.1016/j.enbuild.2016.05.028

[61] Z. Wang, Y. Wang, and R. S. Srinivasan, "A novel ensemble learning approach to support building energy use prediction," *Energy and Buildings*, vol. 159, pp. 109–122, 2018 [Online]. Available: http://dx.doi.org/10.1016/j.enbuild.2017.10.085

[62] F. Ruelens, B. J. Claessens, S. Quaiyum, B. De Schutter, R. Babuška, and R. Belmans, "Reinforcement learning applied to an electric water heater: From theory to practice," *IEEE Transactions on Smart Grid*, vol. 9, no. 4, pp. 3792–3800, 2018.

[63] G. Tardioli, R. Kerrigan, M. Oates, J. O'Donnell, and D. Finn, "Data driven approaches for prediction of building energy consumption at urban level," *Energy Procedia*, vol. 78, pp. 3378–3383, 2015 [Online]. Available: http://dx.doi.org/10.1016/j.egypro.2015.11.754

[64] K. Amasyali and N. M. El-Gohary, "A review of data-driven building energy consumption prediction studies," *Renewable and Sustainable Energy Reviews*, vol. 81, pp. 1192–1205, 2018 [Online]. Available: http://dx.doi.org/10.1016/j.rser.2017.04.095

[65] F. Magoulès, *Data Mining and Machine Learning in Building Energy Analysis*. John Wiley & Sons, 2016.

[66] M. D. Felice, "Short-Term Load Forecasting with Neural Network Ensembles : A Comparative Study," *IEEE computational intelligence magazine*, no. 6, pp. 47–56, 2011.

[67] P. Kepplinger, G. Huber, M. Preißinger, and J. Petrasch, "State estimation of resistive domestic hot water heaters in arbitrary operation modes for demand side management," *Thermal Science and Engineering Progress*, vol. 9, pp. 94–109, 2019 [Online]. Available: `https://doi.org/10.1016/j.tsep.2018.11.003`

[68] H. Braas, U. Jordan, I. Best, J. Orozaliev, and K. Vajen, "District heating load profiles for domestic hot water preparation with realistic simultaneity using DHWcalc and TRNSYS," *Energy*, vol. 201, p. 117552, 2020 [Online]. Available: `https://doi.org/10.1016/j.energy.2020.117552`

[69] L. Kong and N. Zhu, "CFD simulations of thermal stratification heat storage water tank with an inside cylinder with openings," *Procedia Engineering*, vol. 146, pp. 394–399, 2016.

[70] W. Yaïci, M. Ghorab, E. Entchev, and S. Hayden, "Three-dimensional unsteady CFD simulations of a thermal storage tank performance for optimum design," *Applied Thermal Engineering*, vol. 60, pp. 152–163, 2013 [Online]. Available: `http://dx.doi.org/10.1016/j.applthermaleng.2013.07.001`

[71] M. Heleno *et al.*, "Optimizing PV self-consumption through electric water heater modeling and scheduling," in *IEEE Eindhoven PowerTech*, 2015, pp. 1–6.

[72] D. Parra, G. S. Walker, and M. Gillott, "Are batteries the optimum PV-coupled energy storage for dwellings? Techno-economic comparison with water tanks in the UK," *Energy Build*, pp. 614–621, 2016.

[73] M. J. Booysen and M. Roux, "Use of smart grid technology to compare regions and days of the week in household water heating," *International Conference on the Domestic Use of Energy (DUE)*, 2017.

[74] P. J. C. Nel, M. J. Booysen, and B. Van Der Merwe, "A computationally inexpensive energy model for horizontal electric water heaters with scheduling," *IEEE Transactions on Smart Grid*, vol. 9, no. 1, pp. 48–56, 2018.

[75] U. Jordan and K. Vajen, "Realistic domestic hot-water profiles in different time scales," *Report for Solar Heating and Cooling Program of the International Energy Agency (IEA-SHC)Task*, vol. 26, pp. 1–18, 2001 [Online]. Available: `http://sel.me.wisc.edu/trnsys/trnlib/iea-shc-task26/iea-shc-task26-load-profiles-description-jordan.pdf`

[76] R Core Team, *R: A language and environment for statistical computing*. Vienna, Austria: R Foundation for Statistical Computing, 2019 [Online]. Available: `https://www.R-project.org/`

[77] B. Anderson and D. Eyers, *GREENGridData: Processing NZ GREEN Grid project data to create a "safe" version for data archiving and re-use*. 2018 [Online]. Available: `https://github.com/CfSOtago/GREENGridData`

[78] H. Wickham, R. François, L. Henry, and K. Müller, *Dplyr: A grammar of*

data manipulation. 2019 [Online]. Available: https://CRAN.R-project.org/package=dplyr

[79] M. Dowle and A. Srinivasan, *Data.table: Extension of "data.frame"*. 2019 [Online]. Available: https://CRAN.R-project.org/package=data.table

[80] V. Spinu, G. Grolemund, and H. Wickham, *Lubridate: Make dealing with dates a little easier*. 2018 [Online]. Available: https://CRAN.R-project.org/package=lubridate

[81] R. Hyndman *et al.*, *Forecast: Forecasting functions for time series and linear models*. 2019 [Online]. Available: https://CRAN.R-project.org/package=forecast

[82] J. A. Ryan and J. M. Ulrich, *Xts: EXtensible time series*. 2018 [Online]. Available: https://CRAN.R-project.org/package=xts

[83] H. Wickham *et al.*, *Ggplot2: Create elegant data visualisations using the grammar of graphics*. 2019 [Online]. Available: https://CRAN.R-project.org/package=ggplot2

[84] P. J. Aphalo, *Ggpmisc: Miscellaneous extensions to 'ggplot2'*. 2019 [Online]. Available: https://CRAN.R-project.org/package=ggpmisc

[85] B. Auguie, *GridExtra: Miscellaneous functions for "grid" graphics*. 2017 [Online]. Available: https://CRAN.R-project.org/package=gridExtra

[86] Y. Xie, *Knitr: A general-purpose package for dynamic report generation in r*. 2019 [Online]. Available: https://CRAN.R-project.org/package=knitr

[87] G. Daróczi and R. Tsegelskyi, *Pander: An r 'pandoc' writer*. 2018 [Online]. Available: https://CRAN.R-project.org/package=pander

[88] H. Zhu, *KableExtra: Construct complex table with 'kable' and pipe syntax*. 2019 [Online]. Available: https://CRAN.R-project.org/package=kableExtra

[89] D. Meyer, E. Dimitriadou, K. Hornik, A. Weingessel, and F. Leisch, *E1071: Misc functions of the department of statistics, probability theory group (formerly: E1071), tu wien*. 2019 [Online]. Available: https://CRAN.R-project.org/package=e1071

[90] D. G. Ocampo, "Developing an energy-related Time-Use Diary for gaining insights into New Zealand households' electricity consumption," 2015 [Online]. Available: https://ourarchive.otago.ac.nz/bitstream/handle/10523/5957/GiraldoDiana2015MSc.pdf

[91] K. Suomalainen, M. Jack, D. Byers, R. Ford, and J. Stephenson, "Comparative analysis of monitored and self-reported data on electricity use," *Conference Proceedings - 2017 17th IEEE International Conference on Environment and Electrical Engineering and 2017 1st IEEE Industrial and Commercial Power Systems Europe, EEEIC / I and CPS Europe 2017*, pp. 1–4, 2017.

[92] M. Jack and K. Suomalainen, “Potential future changes to residential electricity load profiles – findings from the GridSpy dataset,” *Supplementary report for a presentation at the GREEN Grid Conference, Christchurch*, 2018.

[93] J. M. Wooldridge, *Introductory economics: A modern approach*. South-Western CENGAGE Learning, 2009.

[94] R. B. Cleveland, W. S. Cleveland, J. E. McRae, and I. Terpenning, “STL: A Seasonal-Trend Decomposition Procedure Based on Loess,” *Journal of Official Statistics*, vol. 6, pp. 3–73, 1990.

[95] P. Bloomfield, *Fourier analysis of time series: An introduction*. Wiley series in probability and statistics, 2000.

[96] R. Weron, *Modeling and forecasting electricity loads and prices: A statistical approach*, 1st ed. Sussex: John Wiley & Sons Ltd, 2006.

[97] P. J. Brockwell and R. A. Davis, *Time Series: Theory and Methods*, 2nd ed., vol. 27. Springer, 1991 [Online]. Available: `http://www.springerlink.com/index/D7X7KX6772HQ2135.pdf`

[98] H. Akaike, “A New Look at the Statistical Model Identification,” *IEEE Transactions on Automatic Control*, vol. 19, no. 6, pp. 716–723, 1974.

[99] Rob J. Hyndman and Yeasmin Khandakar, “Automatic Time Series Forecasting: The forecast Package for R,” *Journal of Statistical Software*, vol. 27, no. 3, p. 22, 2008 [Online]. Available: `http://www.jstatsoft.org/v27/i03/paper`

[100] A. J. Smola, K.-R. Müller, G. Rätsch, B. Schölkopf, J. Kohlmorgen, and V. Vapnik, *Advances in kernel methods: Support vector learning*. MIT Press, 1999.

[101] S. Ji, “An illustration of the kernel trick in svm.” 2017 [Online]. Available: `https://commons.wikimedia.org/wiki/File:Kernel_trick_idea.svg`

[102] R. E. Fan, P. H. Chen, and C. J. Lin, “Working set selection using second order information for training support vector machines,” *Journal of Machine Learning Research*, vol. 6, pp. 1889–1918, 2005.

[103] G. James, D. Witten, T. Hastie, and R. Tibshirani, *An Introduction to Statistical Learning with Applications in R*, 8th ed. Springer, 2017.

[104] R. Weron, “Electricity price forecasting: A review of the state-of-the-art with a look into the future,” *International Journal of Forecasting*, vol. 30, no. 4, pp. 1030–1081, 2014 [Online]. Available: `http://dx.doi.org/10.1016/j.ijforecast.2014.08.008`

[105] R. Parker, “Smart control scenarios for domestic water heaters in New Zealand,” *The University of Otago*. unpublished dissertation, 2019.